Risk significance importance measures for a networked system

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INTRODUCTION

Purposes

Our aim is to propose a general approach:

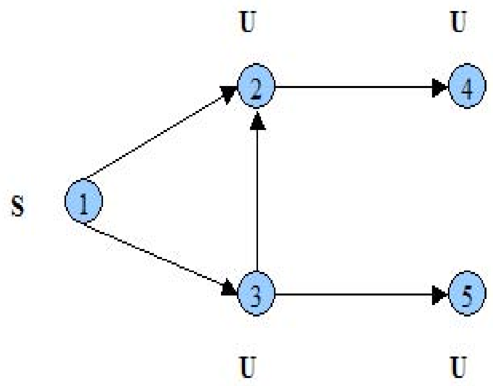
- to assess the Local and Global performances of a networked system,
- to rank its elements basing on a metric referred to the whole network.

Networked system

Random graphs made up of:

unfaultable (Nn) user and (Ns) source nodes,

connected by (Nc) ”directed”, “binary” edges.



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Status of each edge: $x_j = 0$ if the edge is available, 1 elsewhere.

Status of each node: $x_i = 0$ if the node is connected to at least a source trough at least a path made up of available edges, 1 elsewhere.

Local and Global performance of a networked system

The *Local structure function* $x_i(\ldots, x_j, \ldots.)$ is referred to each user node.

<u>Local performance</u>

$$U_i = \Pr\left[x_i(\ldots, x_j, \ldots.) = 1\right] = \sum_{\substack{s=1 \\ s|x_i=1}}^{2^{Nc}} \Pr_s \qquad \textit{where} \quad \Pr_s = \prod_{j=1}^{Nc} (U_j)^{x_j^s} (1 - U_j)^{(1-x_j^s)}$$

The *Global structure function* $\Phi = \Phi(\ldots, x_i(\ldots, x_j, \ldots.), \ldots)$ is referred to the whole network.

A weight w_i is assigned to each user node according to the disutility produced when it is not connected to a source node.

A weight $w_s = \sum_{i=1}^{Nn} (w_i)^{x_i^s}$ results for each status of the system; it is a measure of the total amount of disutility for that status.

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<u>Global performance</u>

$$U = \sum_{s=1}^{2^{Nc}} w_s \cdot \Pr_s = \sum_{i=1}^{Nn} w_i \cdot U_i.$$

If $w_i = 1$ for each user node,

- the weight for the status s of the system is the number of unconnected nodes,
- the Global performance is the sum of the Local performances of the network.

It can result $U > 1$: U is a risk metric for the disutility of the system.

If $w_i = 1/Nn$, the (normalized) metric is less or equal to 1.

If $w_i = N_i / \sum_{i=1}^{Nn} N_i$, where N_i is the number of users relevant to the node i,

the unavailability of the system is the *System Average Interruption Duration Index*

$$SAIDI = \frac{\sum_{i=1}^{Nn} N_i U_i}{\sum_{i=1}^{Nn} N_i}$$

Differential Importance Measure (DIM) for a networked system

Basing on its general definition, DIM results:

$$DIM_j^l = \frac{\dfrac{\partial U_l}{\partial U_j}}{\sum_{j=1}^{Nc} \dfrac{\partial U_l}{\partial U_j}}$$

DIM is additive for a set of edges

$$DIM_{j,k}^i = DIM_j^i + DIM_k^i$$

DIM is not additive for a set of nodes

$$DIM_j^{l,m} \neq DIM_j^l + DIM_j^m$$

For networked system, DIM must be referred to its Global performance.

Basing on $U = \sum_{i=1}^{Nn} w_i \cdot U_i$, DIM referred to the whole network results:

$$*DIM_j = \frac{\dfrac{\partial U}{\partial U_j}}{\sum_{j=1}^{Nc} \dfrac{\partial U}{\partial U_j}} = \frac{\sum_{i=1}^{Nn} w_i \dfrac{\partial U_i}{\partial U_j}}{\sum_{j=1}^{Nc} \sum_{i=1}^{Nn} w_i \dfrac{\partial U_i}{\partial U_j}}$$

Therefore, DIM referred to a well defined user node results:

$$*DIM_j^l = \frac{w_l \cdot \dfrac{\partial U_l}{\partial U_j}}{\sum_{j=1}^{Nc} \sum_{i=1}^{Nn} w_i \cdot \dfrac{\partial U_i}{\partial U_j}}$$

*DIM is additive for a set of edges

$$*DIM_{j,k}^i = *DIM_j^i + *DIM_k^i$$

and for a set of nodes

$$*DIM_j = \sum_{l=1}^{Nn} *DIM_j^l$$

Let us assume that all the edges have the same unavailability $U_j = 0{,}5$.

The probability that the system is in its status s is the same for all statuses.

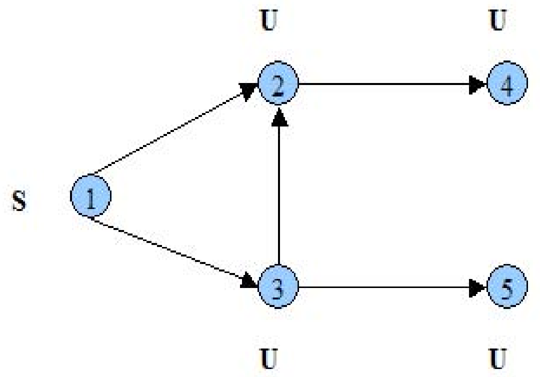
By assuming $w_i = 1$ for each user node, the weight $\left(w_s\right)$ is the number of unconnected nodes $\left(Nn_s\right)$ for the status s.

*DIM results:

$$*DIM_j = \frac{\left(\sum_{\substack{s=1 \\ s|x_j^s=1}}^{2^{Nc}} w_s - \sum_{\substack{s=1 \\ s|x_j^s=0}}^{2^{Nc}} w_s \right)}{\sum_{m=1}^{Nc} \left(\sum_{\substack{s=1 \\ s|x_m^s=1}}^{2^{Nc}} w_s - \sum_{\substack{s=1 \\ s|x_m^s=0}}^{2^{Nc}} w_s \right)} = \frac{\left(\sum_{\substack{s=1 \\ s|x_j^s=1}}^{2^{Nc}} Nn_s - \sum_{\substack{s=1 \\ s|x_j^s=0}}^{2^{Nc}} Nn_s \right)}{\sum_{m=1}^{Nc} \left(\sum_{\substack{s=1 \\ s|x_m^s=1}}^{2^{Nc}} Nn_s - \sum_{\substack{s=1 \\ s|x_m^s=0}}^{2^{Nc}} Nn_s \right)}$$

*DIM can be computed basing on the enumeration of the statuses of the system.

APPLICATION CASE



Analytical results

$$U_2 = U_{21} \cdot (U_{23} + U_{31} - U_{23} \cdot U_{31}) = 0{,}3750$$

$$U_3 = U_{31} = 0{,}5000$$

$$U_4 = U_{42} + U_{21} \cdot (U_{23} + U_{31} - U_{23} \cdot U_{31}) - U_{42} \cdot U_{21} \cdot (U_{23} + U_{31} - U_{23} \cdot U_{31}) = 0{,}6875$$

$$U_5 = U_{53} + U_{31} - U_{53} \cdot U_{31} = 0{,}7500$$

Enumeration of the system statuses

System status	Edges status					Nodes status				$w_s = N_s$
	21	31	23	42	53	2	3	4	5	
1	0	0	1	1	1	0	0	1	1	**2**
...	...	...	...	...	...	...	...	...	...	**...**
32	1	1	0	0	0	1	1	1	1	**4**
Total	**-**	**-**	**-**	**-**	**-**	**12**	**16**	**22**	**24**	**74**

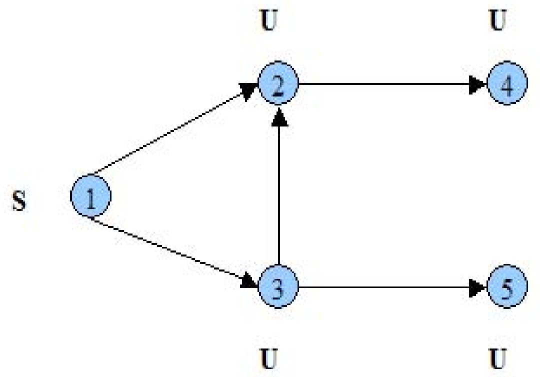
$$\sum_{s=1}^{2^{Nc}} w_s = \sum_{s=1}^{2^{Nc}} N_s = 74$$

$$\Pr_s = \frac{1}{2^5} = \frac{1}{32}$$

$$U = \frac{12}{32} + \frac{16}{32} + \frac{22}{32} + \frac{24}{32} = \frac{74}{32}$$

$$= 0{,}3750 + 0{,}5000 + 0{,}6875 + 0{,}7500 = 2{,}315$$

Count of the system statuses

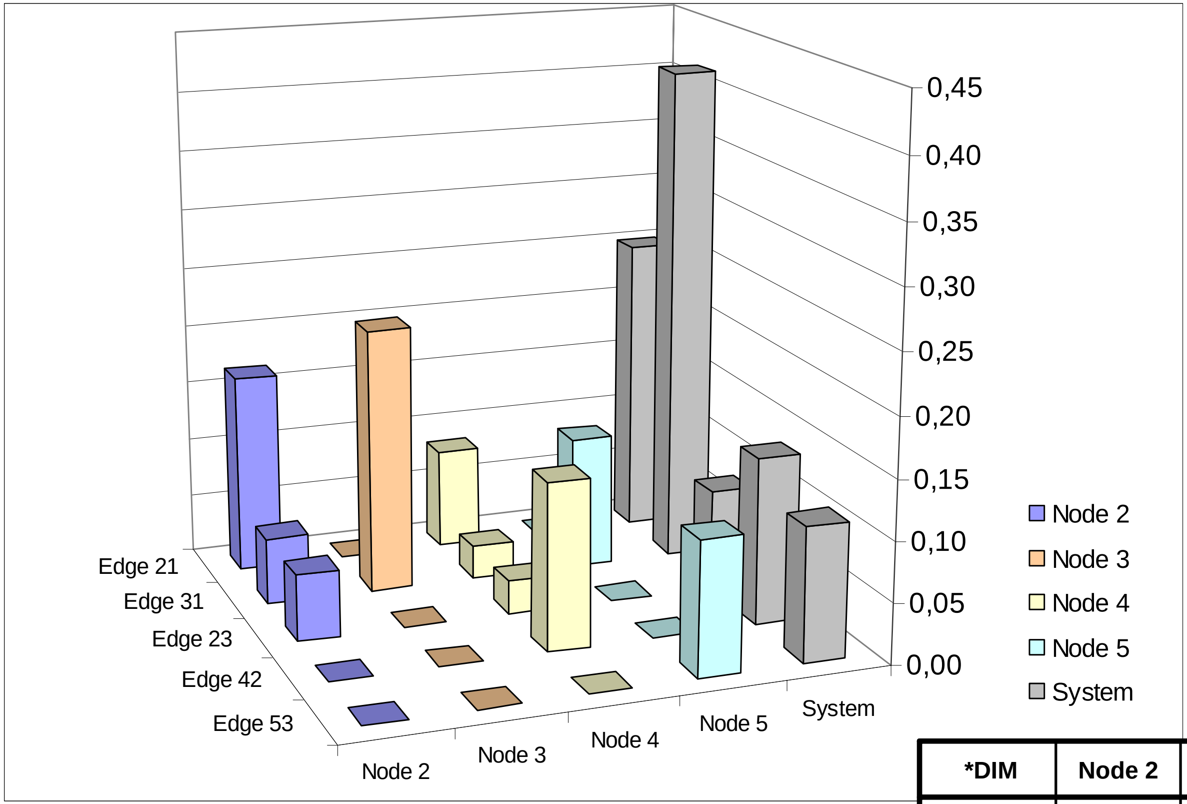


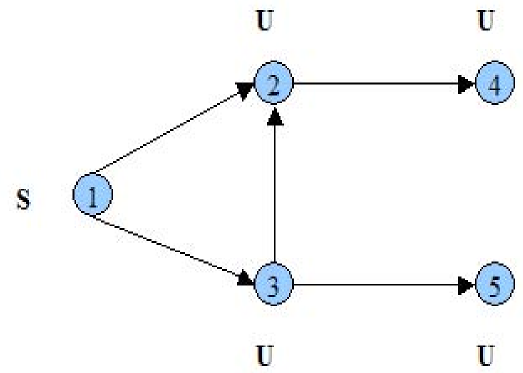
In order to compute *DIM

- we consider one edge at time,
- we analyze the structure of the network when the edge is always available or failed,
- we evaluate the sum over the system's statuses of the number of unconnected nodes or, in the same way, the number of statuses for which each node is not connected to a source and their sum.

Edge		Number of system failure statuses				
ID	**Status**	**Node 2**	**Node 3**	**Node 4**	**Node 5**	**Total**
Edge 21	Available	0	8	8	12	**28**
	Not available	12	8	14	12	**46**
…	…	…	…	…	…	**…**

*Differential Importance Measure

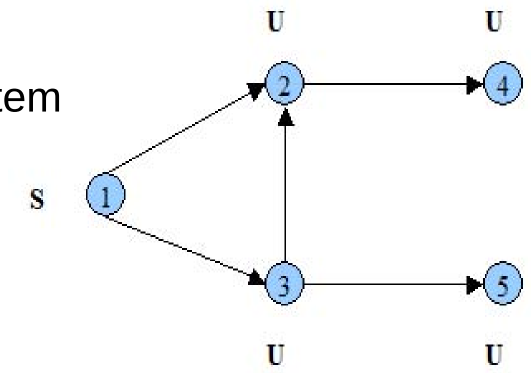




$$* DIM_j = \frac{\left(\sum_{\substack{s=1 \\ s | x_j^s = 1}}^{2^{Nc}} Nn_s - \sum_{\substack{s=1 \\ s | x_j^s = 0}}^{2^{Nc}} Nn_s \right)}{\sum_{m=1}^{Nc} \left(\sum_{\substack{s=1 \\ s | x_m^s = 1}}^{2^{Nc}} Nn_s - \sum_{\substack{s=1 \\ s | x_m^s = 0}}^{2^{Nc}} Nn_s \right)}$$

*DIM	Node 2	Node 3	Node 4	Node 5	System
Edge 21	0,17	0,00	0,08	0,00	0,25
Edge 31	0,06	0,22	0,03	0,11	0,42
Edge 23	0,06	0,00	0,03	0,00	0,08
Edge 42	0,00	0,00	0,14	0,00	**0,14**
Edge 53	0,00	0,00	0,00	0,11	**0,11**
Total	0,28	0,22	0,28	0,22	1,00

DIM and *DIM rank in a different way the elements of the system



DIM	Node 2	Node 3	Node 4	Node 5	System
Edge 21	0,60	0,00	0,30	0,00	0,90
Edge 31	0,20	1,00	0,10	0,50	1,80
Edge 23	0,20	0,00	0,10	0,00	0,30
Edge 42	0,00	0,00	0,50	0,00	**0,50**
Edge 53	0,00	0,00	0,00	0,50	**0,50**
Total	1,00	1,00	1,00	1,00	4,00

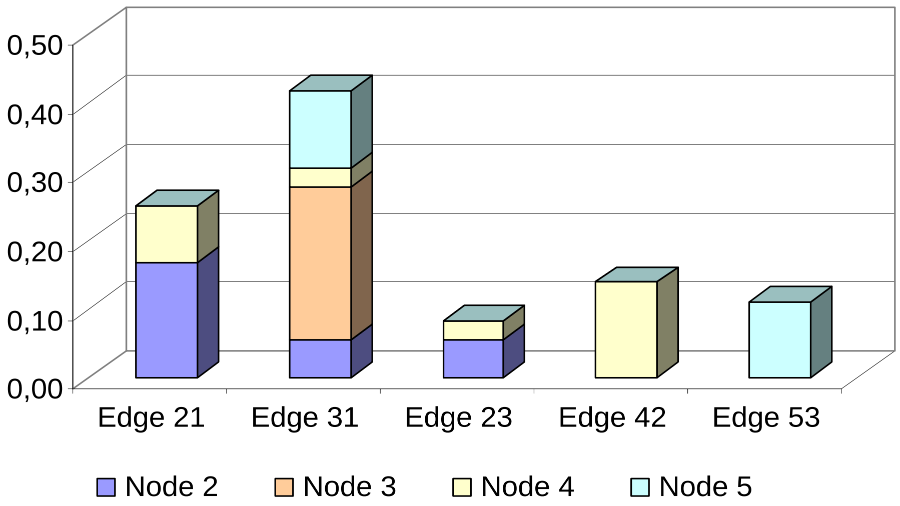
*DIM	Node 2	Node 3	Node 4	Node 5	System
Edge 21	0,17	0,00	0,08	0,00	0,25
Edge 31	0,06	0,22	0,03	0,11	0,42
Edge 23	0,06	0,00	0,03	0,00	0,08
Edge 42	0,00	0,00	0,14	0,00	**0,14**
Edge 53	0,00	0,00	0,00	0,11	**0,11**
Total	0,28	0,22	0,28	0,22	1,00

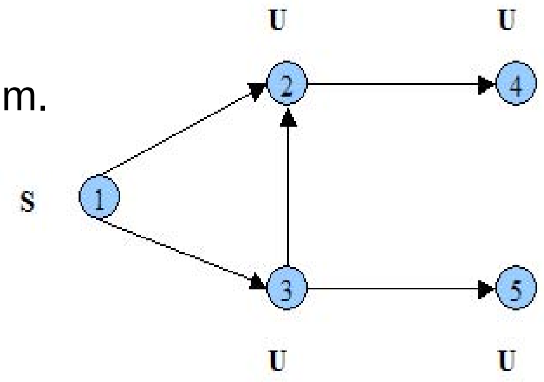
Remark:

According to *DIM, the edge 42 is more important than the edge 53 because the unavailability of the node 2 is less than the unavailability of the node 3 (redundant paths).

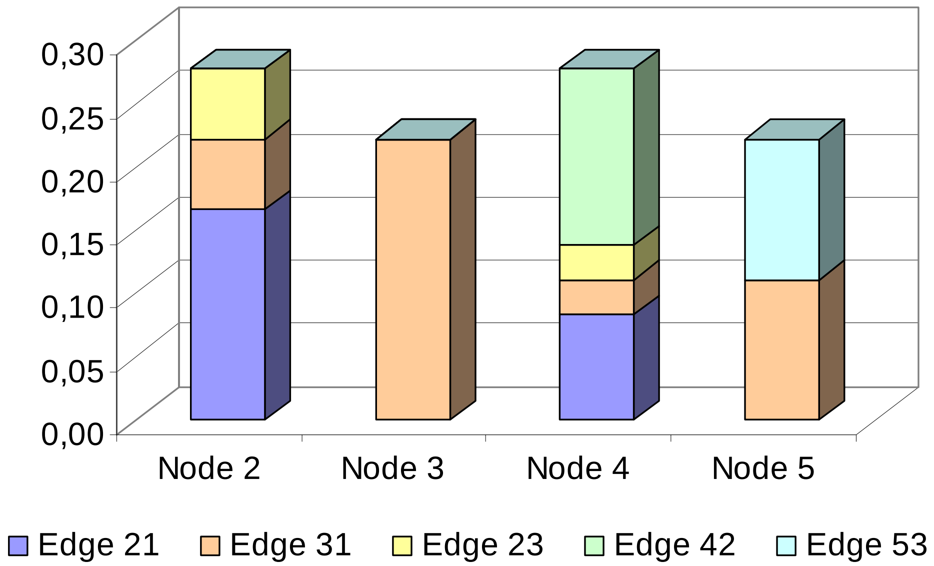
Different results (the same importance for both the edges) are obtained by DIM.

The sum of *DIM referred to the same edge provides its importance with respect to the Global performance of the system.





The sum of *DIM referred to the same node provides a ranking of nodes themselves.



CONCLUSION

The availability / reliability performance of a networked system can be assessed with respect to each user node (Local performances) and to the whole network (Global performances).

The structure of a networked system can be assessed effectively by assuming the same unavailability for all edges and the same weight for all user nodes and ranking them by means of an adequate measure.

The elements of a networked system can be ranked by means of an additive importance measure (Differential importance measure) which is referred to its Global performance.

The same approach can be used assuming different values for the edges unavailability and/or for the weights assigned to each user node.

Further studies (see "*Reliability assessment basing on importance measure*"):

- evaluation of the Local and Global performance of a networked system by means of Monte Carlo simulation, without the identification of the system structure functions and the enumeration of its statuses;
- adoption of different importance measures, to refer to the Global performance of the system.

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