New Results on the Differential Importance Measures of Markovian Systems

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Background

- Reliability importance measure is a powerful tool to identify which components contribute the most to system (un) performance, and suggests optimal modification for system upgrade (design improvement, better maintenance,...)
- Recently, a new importance measure, called Differential Importance Measure (DIM), has been introduced for use in risk-informed decision-making.

Purposes:

Develop this importance measure in the context of dynamic systems including inter-component, functional dependencies, or more generally, systems described by Markov models.

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Outline

- Definition
- First-order approach
- High-order approach
- Numerical example
- Conclusions/future works

Main idea:

Consider a n-components dynamic system described by a Markov model with:

- transition rates matrix **M**,
- vector of steady state probabilities π,
- system availability at the steady-state $A = \pi \boldsymbol{f}$.

Problems:

- evaluate the total variation of the system availability due to the change of N parameters of the system (e.g., failure and/or repair rates),
- identify the relative contribution of one component or of a group of components on this total variation

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Main concept:

- The variation of the system availability provoked by the change, $x_i \rightarrow x_i + \Delta x_i$:

$$\Delta A_{x_i} = \Delta x_i \frac{\partial A}{\partial x_i} + \frac{1}{2!}(\Delta x_i \frac{\partial}{\partial x_i})^2 A + \frac{1}{3!}(\Delta x_i \frac{\partial}{\partial x_i})^3 A + \ldots$$

- The total variation due to the simultaneous changes of N parameters:

$$\Delta A = \sum_{i=1}^{N} \Delta x_i \frac{\partial A}{\partial x_i} + \frac{1}{2!}(\sum_{i=1}^{N} \Delta x_i \frac{\partial}{\partial x_i})^2 A + \frac{1}{3!}(\sum_{i=1}^{N} \Delta x_i \frac{\partial}{\partial x_i})^3 A + \ldots$$
$$= \Delta^I A + \Delta^{II} A + \Delta^{III} A + \ldots,$$

where: $\Delta^k A = \frac{1}{k!}(\sum_{i=1}^{N} \Delta x_i \frac{\partial}{\partial x_i})^k A = \frac{1}{k!}\sum_{i_1=1}^{N}\sum_{i_2=1}^{N}\cdots\sum_{i_k=1}^{N} \Delta x_{i_1}\Delta x_{i_2}\ldots\Delta x_{i_k} \frac{\partial^k A}{\partial x_{i_1}\partial x_{i_2}\ldots\partial x_{i_k}}$

Differential Importance Measure (DIM) of:

- one parameter (x_i): $\mathrm{DIM}(x_i) = \frac{\Delta A_{x_i}}{\Delta A}$
- a group of parameters ($x_i, x_j, \ldots, x_k$): $\mathrm{DIM}(x_i, x_j, \ldots, x_k) = \frac{\Delta A_{x_i, x_j, \ldots, x_k}}{\Delta A}$

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- If the changes of parameters are *small enough*:
  - $$\delta A_{x_i} \simeq \delta^I A_{x_i} = \delta x_i \frac{\partial A}{\partial x_i} = \delta x_i \frac{\partial \pi}{\partial x_i} \boldsymbol{f}$$
  - $$\delta A \simeq \delta^I A = \sum_{j=1}^{N} \delta x_j \frac{\partial A}{\partial x_j} = \sum_{j=1}^{N} \delta^I A_{x_j}.$$
- In the framework of the steady-state markov models, $\pi \mathbf{M} = 0$, therefore:

$$\frac{\partial \pi}{\partial x_i}\mathbf{M} + \pi \frac{\partial \mathbf{M}}{\partial x_i} = 0 \Rightarrow \frac{\partial \pi}{\partial x_i} = -\pi \mathbf{Q}_{x_i} \mathbf{M}^\sharp$$

(where: $\mathbf{Q}_{x_i} = \frac{\partial \mathbf{M}}{\partial x_i}$ and $\mathbf{M}^\sharp = (\mathbf{M} + \boldsymbol{e}\pi)^{-1} - \boldsymbol{e}\pi$, $\boldsymbol{e} = (1, 1, \ldots, 1)^T$, is the groupe inverse of $\mathbf{M}$)

Consequently:

$$\delta^I A_{x_i} = \delta x_i \frac{\partial \pi}{\partial x_i} \boldsymbol{f} = -\pi \delta x_i \mathbf{Q}_{x_i} \mathbf{M}^\sharp = -\pi \mathbf{Q}_{\delta x_i} \mathbf{M}^\sharp \boldsymbol{f}, \text{ with } \mathbf{Q}_{\delta x_i} = \delta x_i \mathbf{Q}_{x_i}$$

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- First-order differential importance measure:

$$\mathrm{DIM}^I(x_i) = \frac{\delta^I A_{x_i}}{\delta^I A} = \frac{\boldsymbol{\pi}\mathbf{Q}_{\delta x_i}\mathbf{M}^{\sharp}\boldsymbol{f}}{\boldsymbol{\pi}\mathbf{Q}_{\delta}\mathbf{M}^{\sharp}\boldsymbol{f}},$$

- DIM^I of a group of parameters (components):

$$\mathrm{DIM}^I(x_i, x_j, ..., x_s) = \frac{\boldsymbol{\pi}\mathbf{Q}_{\delta x_i, \delta x_j, \ldots, \delta x_s}\mathbf{M}^{\sharp}\boldsymbol{f}}{\boldsymbol{\pi}\mathbf{Q}_{\delta}\mathbf{M}^{\sharp}\boldsymbol{f}} = \mathrm{DIM}^I(x_i) + \mathrm{DIM}^I(x_j) + ... + \mathrm{DIM}^I(x_s)$$

Generalized DIM^I:

- In the framework of markov model:

$$\begin{cases} \mathbf{M}_{\mathbf{Q}_i} & = \mathbf{M} + \delta_i \mathbf{Q}_i, \text{ variation on a specific direction} \\ \mathbf{M}_{\mathbf{Q}} & = \mathbf{M} + \sum_{i=1}^{N} \delta_i \mathbf{Q}_i, \text{ simulneous variations of } N \text{ different directions} \end{cases}$$

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Remarks:

Advantages:

- simple, easy to evaluate,
- DIM^I is additive.

disadvantages:

- DIM^I does not account for the effects of simultaneous changes of several parameters
- DIM^I is only applicable when the changes of parameters verify the small enough conditions.

Solution:

It is interesting to take into account the higher-order effects.

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Solution:

It is interesting to take into account the higher-order effects.

By taking the second-order derivative, we obtain:

- $\Delta^{II}A = \pi(\mathbf{Q}_{\Delta}\mathbf{M}^{\sharp})^2\boldsymbol{f}$
- $\Delta^{II}A_{x_i} = \pi(\mathbf{Q}_{\Delta x_i}\mathbf{M}^{\sharp})^2\boldsymbol{f}$

Similarly, by taking the higher-order derivatives:

- $\Delta^{h}A = (-1)^h\pi(\mathbf{Q}_{\Delta}\mathbf{M}^{\sharp})^h\boldsymbol{f}$, with $h = 3, 4, \ldots$
- $\Delta^{h}A_{x_i} = (-1)^h\pi(\mathbf{Q}_{\Delta x_i}\mathbf{M}^{\sharp})^h\boldsymbol{f}$

Consequently:

$$\Delta A = \Delta^{I}A + \Delta^{II}A + \ldots + \Delta^{h}A + \ldots = \sum_{i=1}^{\infty}\Delta^{i}A = \sum_{i=1}^{\infty}\pi\left[(-1)^i\mathbf{Q}_{\Delta}\mathbf{M}^{\sharp}\right]^i\boldsymbol{f}$$

$$\Delta A_{x_i} = \Delta^{I}A_{x_i} + \Delta^{II}A_{x_i} + \ldots + \Delta^{h}A_{x_i} + \ldots = \sum_{i=1}^{\infty}\pi\left[(-1)^i\mathbf{Q}_{\Delta x_i}\mathbf{M}^{\sharp}\right]^i\boldsymbol{f}$$

- DIM:

$$\mathrm{DIM}(x_i) = \frac{\Delta A_{x_i}}{\Delta A} = \frac{\sum_{i=1}^{\infty} \boldsymbol{\pi}\left[(-1)^i \mathbf{Q}_{\Delta x_i} \mathbf{M}^{\sharp}\right]^i \boldsymbol{f}}{\sum_{i=1}^{\infty} \boldsymbol{\pi}\left[(-1)^i \mathbf{Q}_{\Delta} \mathbf{M}^{\sharp}\right]^i \boldsymbol{f}}$$

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$$\mathrm{DIM}^h(x_i) = \frac{\sum_{i=1}^{h} \boldsymbol{\pi}\left[(-1)^i \mathbf{Q}_{\Delta x_i} \mathbf{M}^{\sharp}\right]^i \boldsymbol{f}}{\sum_{i=1}^{h} \boldsymbol{\pi}\left[(-1)^i \mathbf{Q}_{\Delta} \mathbf{M}^{\sharp}\right]^i \boldsymbol{f}}$$

Generalized DIM

$$\mathrm{DIM}(\mathbf{Q}_i) = \frac{\Delta A_{\mathbf{Q}_i}}{\Delta A} = \frac{\sum_{i=1}^{\infty} \pi\left[(-1)^i \Delta_i \mathbf{Q}_i \mathbf{M}^{\sharp}\right]^i \boldsymbol{f}}{\sum_{i=1}^{\infty} \pi\left[(-1)^i (\sum_{i=1}^{N} \Delta_i \mathbf{Q}_i) \mathbf{M}^{\sharp}\right]^i \boldsymbol{f}}$$

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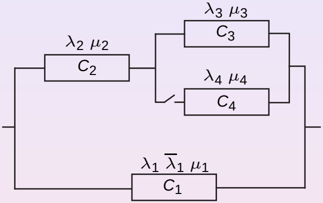
Consider a dynamic system with 4 dependent components (cold spare[C_3,C_4] & shared load[C_1,group(C_2, C_3, C_4)])

Unit	λ_i	μ_i	$\overline{\lambda}_i$
C_1	2e-3	1e-3	2.5e-3
C_2	1e-3	9e-3	-
C_3	2e-4	2.9e-3	-
C_4	3e-4	6e-3	-



Assumption:

Assume that all components' failure rates are changed simultaneously
(for example, all of the component in an aircraft would presumably be subjected to many of the same stresses vibration from the engines, shock of landing, irregularities in the power supplied, etc.)

Objective:

Identify the important component according to its relative contribution on the total variation of the system availability at the steady-state (using the DIM).

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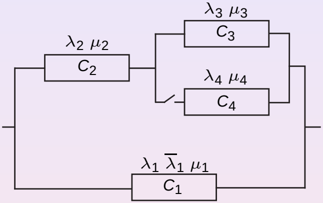

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Scenarios of changes

To illustrate the application of DIMs, three scenarios of changes (uniform percentage changes $\frac{\Delta\lambda_i}{\lambda_i} = \frac{\Delta\lambda_j}{\lambda_j} = \omega$, with $i, j = 1, .., 4$) are proposed: $\omega = 1, 7, 10\%$

- Case $\omega = 1\%$:

	C_1	C_2	C_3	C_4	Order
DIM^{I}	0.222047	0.209410	0.210678	0.357866	$C_4 > C_1 > C_3 > C_2$
DIM^{II}	0.220546	0.209234	0.209637	0.357326	$C_4 > C_1 > C_3 > C_2$
DIM^{III}	0.220560	0.209238	0.209645	0.357333	$C_4 > C_1 > C_3 > C_2$
DIM^{VI}	0.220560	0.209238	0.209645	0.357333	$C_4 > C_1 > C_3 > C_2$

- DIM^{I}, DIM^{II},DIM^{III} and DIM^{VI} can provide the same ranking.

- The most important component is C_4 and C_2 is the less important one.

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- Case $\omega = 7\%$:

	C_1	C_2	C_3	C_4	Order
DIM^{I}	0.222047	0.209410	0.210678	0.357866	$C_4 > C_1 > C_3 > C_2$
DIM^{II}	0.211533	0.208183	0.203384	0.354088	$C_4 > C_1 > C_2 > C_3$
DIM^{III}	0.212220	0.208364	0.203818	0.354426	$C_4 > C_1 > C_2 > C_3$
DIM^{VI}	0.212189	0.208356	0.203801	0.354412	$C_4 > C_1 > C_2 > C_3$

- Case $\omega = 10\%$

	C_1	C_2	C_3	C_4	Order
DIM^{I}	0.222047	0.209410	0.210678	0.357866	$C_4 > C_1 > C_3 > C_2$
DIM^{II}	0.207021	0.207656	0.200254	0.352466	$C_4 > C_2 > C_1 > C_3$
DIM^{III}	0.208418	0.208025	0.201136	0.353156	$C_4 > C_1 > C_2 > C_3$
DIM^{VI}	0.208329	0.208003	0.201088	0.353116	$C_4 > C_1 > C_2 > C_3$

- DIM^{I} of all components' failure rates remains unchange.
- DIM^{II}, DIM^{III} and DIM^{VI}'s values change leading to different rankings.
- DIM^{III} and DIM^{VI} can provide a more precise importance ranking.

- DIM can be used to identify the importance of one component (or a group of components),
- Generalized DIM can be used to identify the importance of a direction of interest,
- DIM^{I} is only applicable when the changes of parameters verify the small enough conditions,
- DIM^{h} would provide better results than those obtained from the DIM^{I} and DIM^{II}, it can be used even the changes of parameters are not small enough,
- The need to resort to information on the high-order effects depends on the magnitude of the change of parameters' values.

Future works:

- Develop methods to find the minimal h for which we can provide the true importance ranking,
- Use the Perturbation Analysis to estimate the DIMs from the operating feedback data.

- DIM can be used to identify the importance of one component (or a group of components),
- Generalized DIM can be used to identify the importance of a direction of interest,
- DIM^{I} is only applicable when the changes of parameters verify the small enough conditions,
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