An Intuitive Method for Reliability Analysis of Dynamic Systems

Seung Ki Shin, Gyoung Tae Goh and Poong Hyun Seong
Korea Advanced Institute of Science and Technology (KAIST)

2008 PSAM9
May 23rd, 2008

Contents

- *Introduction*
- *Reliability Graph with General Gates*
- *dding Dynamic Nodes to RGGG*
- *Probability Tables for Dynamic Nodes*
  - *Discrete-time method*
- *Development of a Tool for Generating PT*
- *Summary & Further Study*
- *References*

Introduction

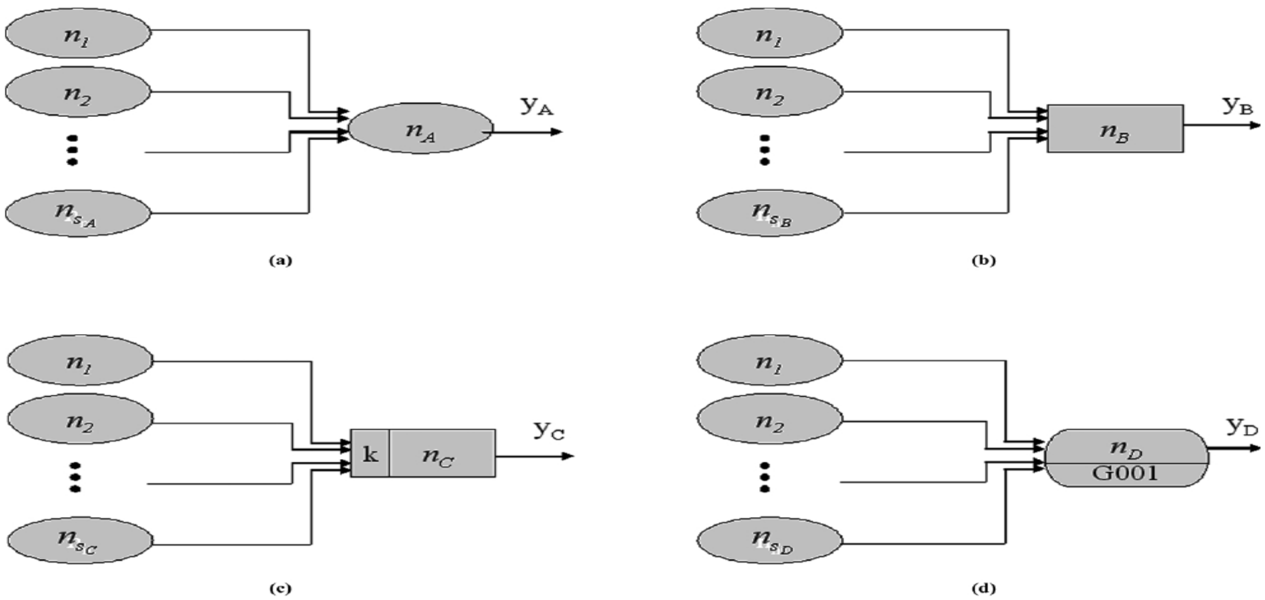
- Among the methods to analyze system reliability, fault tree analysis is the most widely used method, but it's not an intuitive method.
- Reliability graph with general gates (RGGG) method was proposed in order to overcome this shortcoming of fault tree.
- Conventional fault tree and RGGG method cannot capture the dynamic behavior of system associated with time dependent events.
- Dynamic fault tree method was proposed, but it's also not intuitive. And when calculating the reliability from dynamic fault tree, usually Markov chain method is used, but it has the state space explosion problem.
- If it is possible to add a sequential concept to RGGG, it will become an intuitive dynamic method.

Reliability Graph with General Graph

- Reliability graph is an intuitive method, so it can model systems by one-to-one match graph.
- The reason why it's not used widely is low expression power. (Only OR gate)
- RGGG which utilizes general gates was proposed.
- By determining the probability table for each node, RGGG can be transformed to an equivalent Bayesian network and the system reliability can be calculated.

Reliability Graph with General Graph

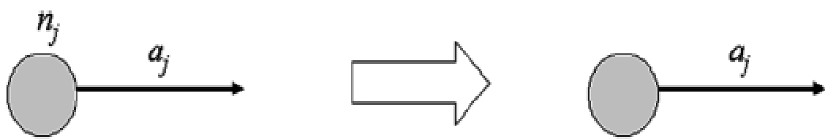
- RGGG nodes



- (a) OR (b) AND (c) k-out-of-n (d) general purpose gate

Reliability Graph with General Graph

- Transformation to reliability graph with perfect node

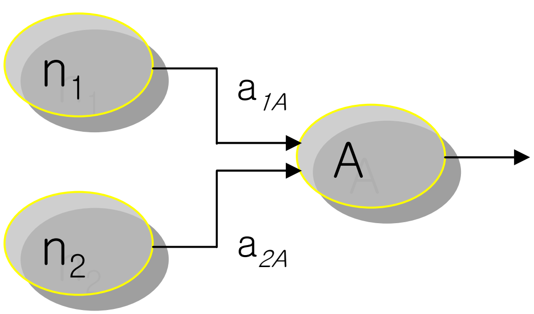


$\Pr\{n_j \text{ as successful}\} = Pn_j$ $\quad\quad$ $\Pr\{n_j \text{ as successful}\} = 1$

$\Pr\{a_j \text{ as successful}\} = Pa_j$ $\quad\quad$ $\Pr\{a_j \text{ as successful}\} = Pn_j \times Pa_j$

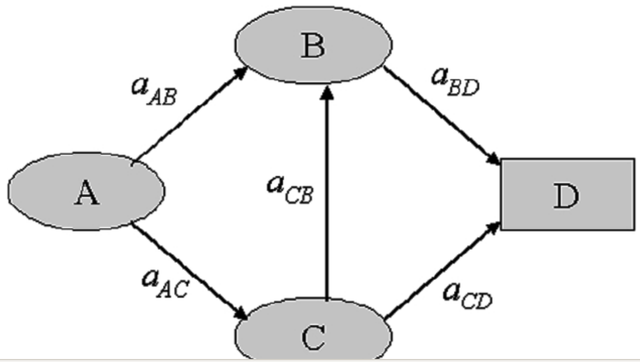
Ex.) Probability table for a node with OR gate when $n = 2$

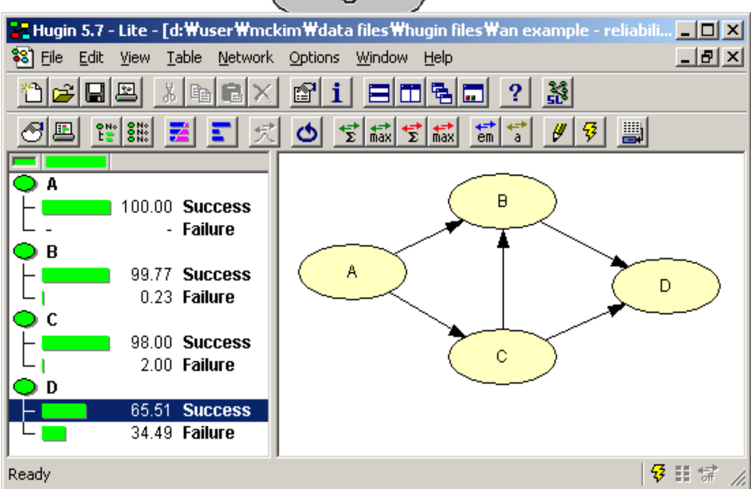
	$y_1 = 1$ (success)		$y_1 = 0$ (failure)	
	$y_2 = 1$ (success)	$y_2 = 0$ (failure)	$y_2 = 1$ (success)	$y_2 = 0$ (failure)
$y_A = 1$ (success)	$r_{1A} + r_{2A} - r_{1A}r_{2A}$	r_{1A}	r_{2A}	0
$y_A = 0$ (failure)	$1 - (r_{1A} + r_{2A} - r_{1A}r_{2A})$	$1 - r_{1A}$	$1 - r_{2A}$	1

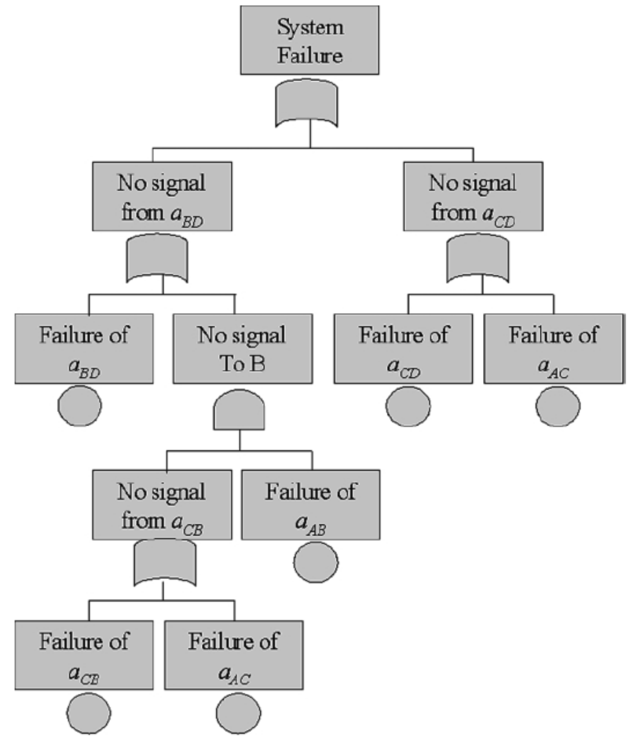


Reliability Graph with General Graph

- Comparison of fault tree and RGGG

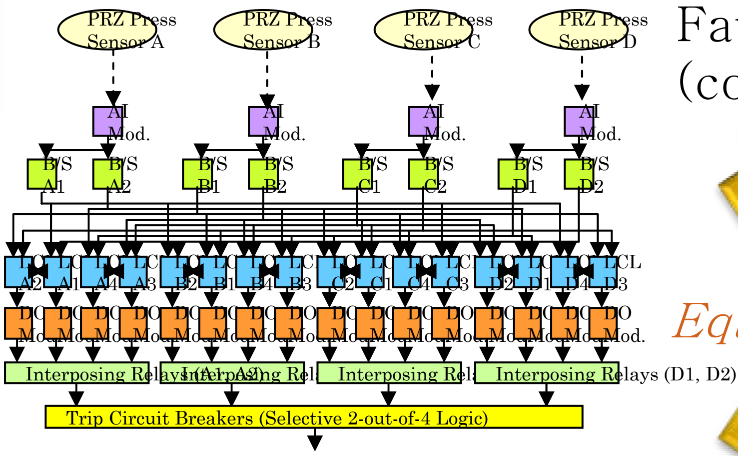






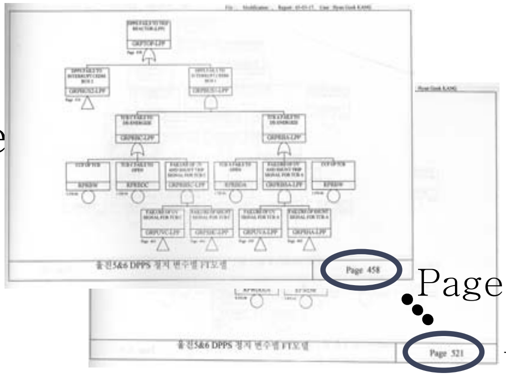
Reliability Graph with General Graph

Ex. 1) Digital Plant Protection System



Structure of DPPS
(For low pressurizer pressure trip)

Fault Tree
(complex)



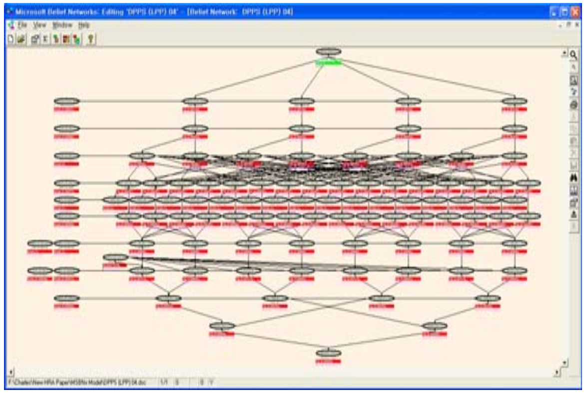
Page 458

Page 521

64 pages !!!

Equivalent !!!

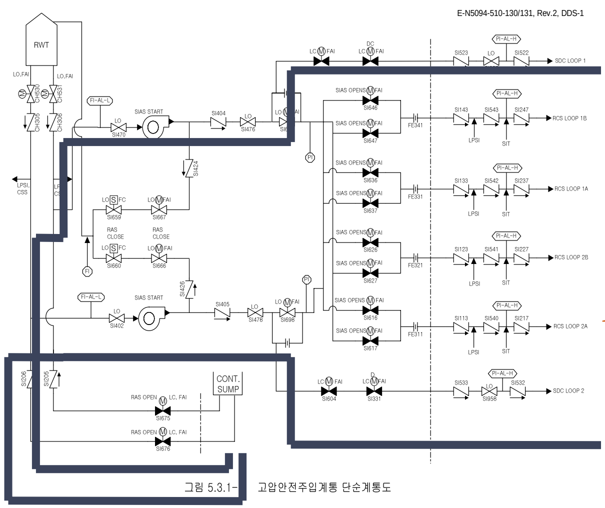
RGGG
(more intuitive)



Only 1 page !!!

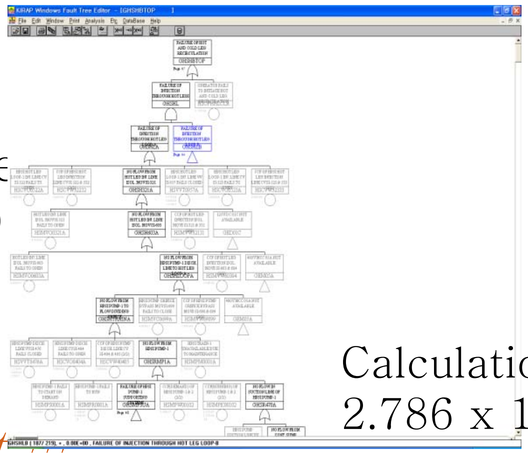
Reliability Graph with General Graph

Ex. 2) Recirculation in KSNPs



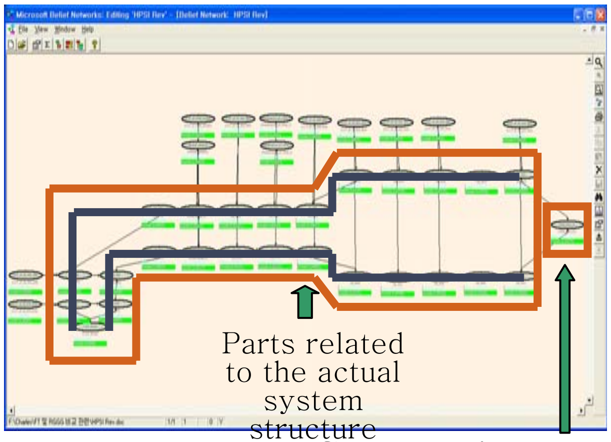
High Pressure
Recirculation
(in KSNP-type NPPs)

Fault Tree
(complex)



Calculation result :
2.786×10^{-3}

Equivalent !!!



RGGG
(more intuitive)

Calculation result :
2.7746×10^{-3}

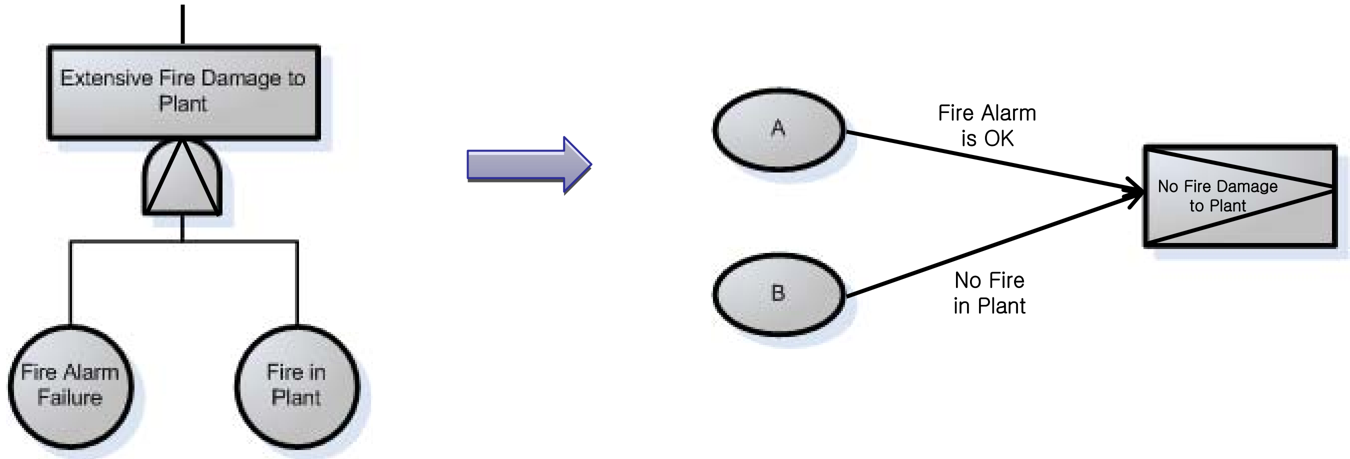
Adding Dynamic Nodes to RGGG

- The existing reliability graph with general gate (RGGG) method cannot capture dynamic behaviors of a system associated with sequence dependent failure events.

- We add dynamic nodes to existing RGGG and conduct quantitative analysis by using discrete-time method.
  → Dynamic RGGG

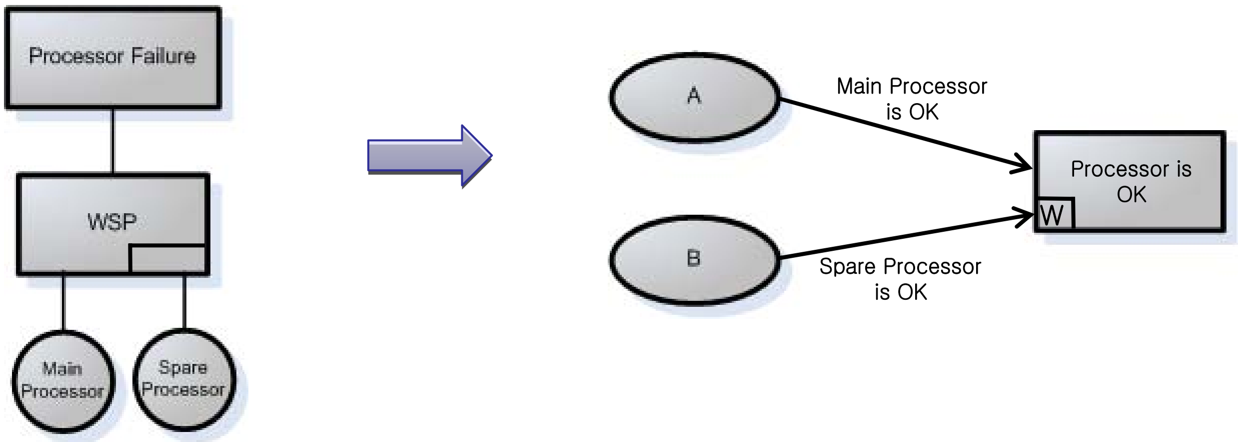
Adding Dynamic Nodes to RGGG

- Priority-AND (PAND) node
  - Node fails only if both signals from A and B are disconnected and a signal from A is disconnected before B.



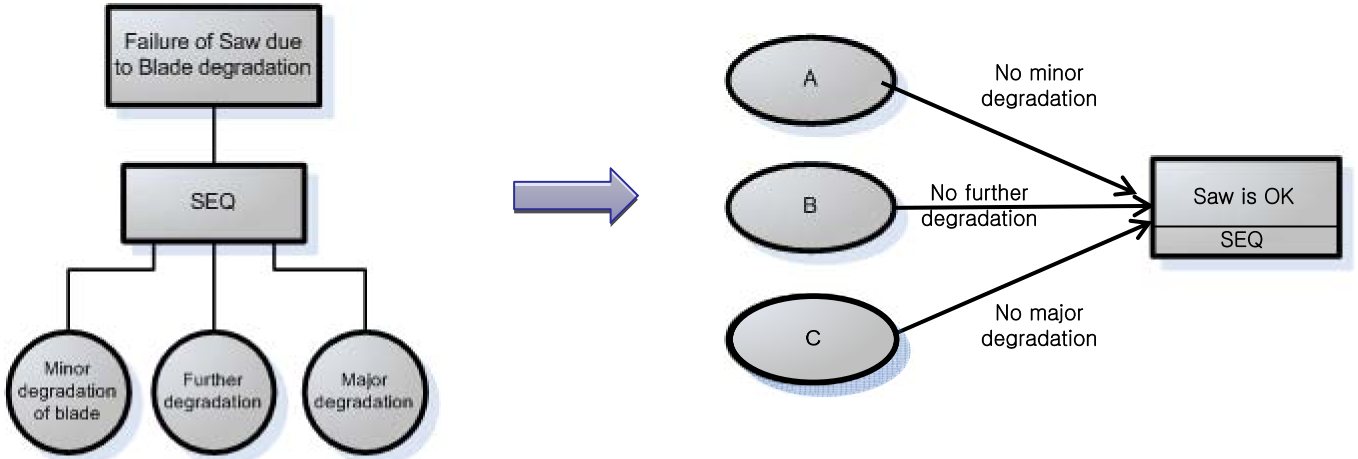
Adding Dynamic Nodes to RGGG

- Spare node
  - Node fails only if a primary signal and all spare signals are disconnected.



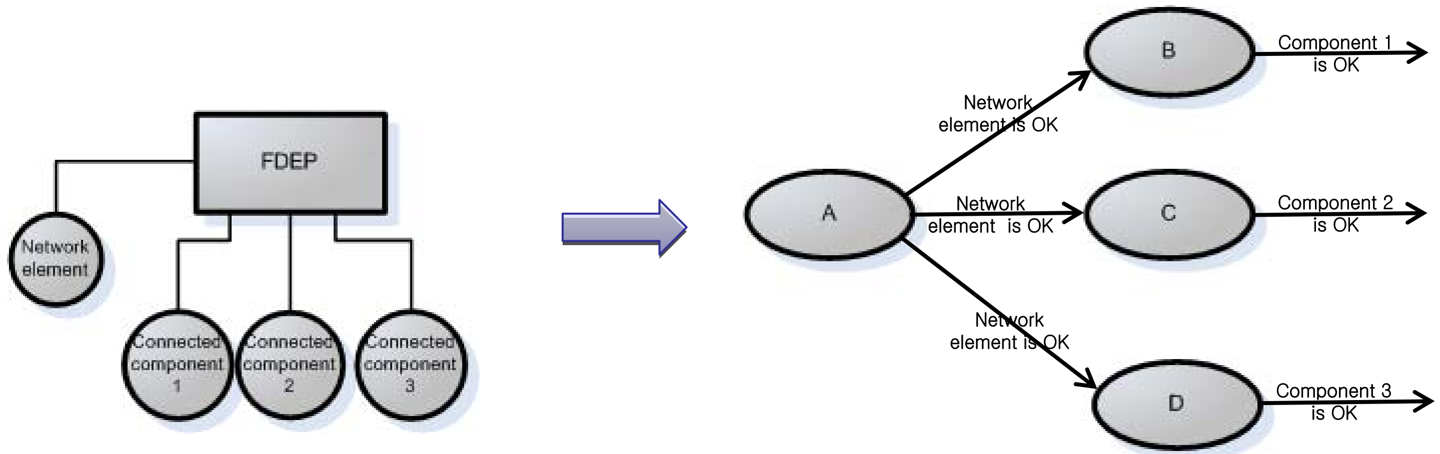
Adding Dynamic Nodes to RGGG

- SEQ node
  - The input signals are constrained to be disconnected in the particular order and SEQ node fails if and only if all input signals are disconnected.



Adding Dynamic Nodes to RGGG

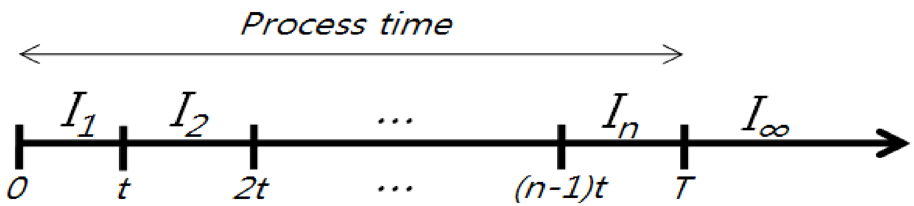
- FDEP node
  - This node is not necessary in RGGG.
  - Existing RGGG can illustrate the property of FDEP gate.



Probability Tables for Dynamic Nodes

- In order to transform a dynamic reliability graph to an equivalent Bayesian network, the probability table corresponding to each dynamic node should be determined.

  - The discrete-time method is used.



$$t = \frac{T}{n}$$

  - Divide the process time line into *n* same intervals.
  - Output of each node is one of $\{I_1, I_2, \ldots, I_n, I_\infty\}$.
    - I_k means that the node is failed in *kth interval*.
    - I_∞ means that the node is never failed.
  - P_{ij}^k : The probability that an arc a_{ij} from node n_i to n_j is failed in the *k*th time interval
    - When the cumulative failure distribution function of a_{ij} is $F_{ij}(t)$,

$$P_{ij}^k = \int_{(k-1)t}^{kt} \frac{dF_{ij}(t)}{dt} dt$$

Probability Tables for Dynamic Nodes

- In order to obtain the accurate reliability, *n* (the number of time discretization) should increases, but as *n* increases, the probability table of each node becomes more complex.
  - The number of blanks that should be filled is $(n+1)^3$ per each node. (2 inputs)

    Ex) $n = 3$,

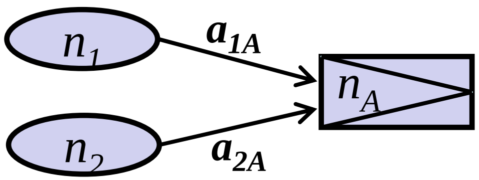
		C			
A	B	l_1	l_2	l_3	l_∞
l_1	l_1				
	l_2				
	l_3				
	l_∞				
l_2	l_1				
	l_2				
	l_3				
	l_∞				
l_3	l_1				
	l_2				
	l_3				
	l_∞				
l_∞	l_1				
	l_2				
	l_3				
	l_∞				

64 blanks should be filled.

  - The rule of making probability tables is necessary to reduce making time and mistakes.

Probability Tables for Dynamic Nodes

- PAND node



- Let output of n_1 , n_2 , n_A as I_x , I_y , I_z
  - I_x means that n_1 fails at x^{th} interval.

$y < z$	• 0
$x \geq z, y = z$	• $P(a_{1A}$ fails before z interval$)$ $\cdot(1 - P(a_{2A}$ fails before z interval$))$
$x \geq z, y > z$	• $P(a_{1A}$ fails before z interval$)$ $\cdot P(a_{2A}$ fails at z interval$)$
$x < z, y = z$	• $1 - P(a_{2A}$ fails before z interval$)$
$x < z, y > z$	• $P(a_{2A}$ fails at z interval$)$

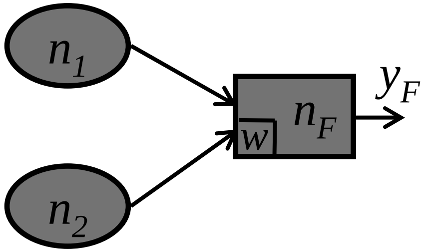
Ex.) $n=3$ $\quad x, y, z \in \{1, 2, 3, \infty\}$

		n_A			
n_1	n_2	I_1	I_2	I_3	I_∞
I_1	I_1	0	0	0	1
	I_2	0	$1 - P_{2A}^1$	0	$1-\Sigma$
	I_3	0	P_{2A}^2	$1 - P_{2A}^1 - P_{2A}^2$	$1-\Sigma$
	I_∞	0	P_{2A}^2	P_{2A}^3	$1-\Sigma$
I_2	I_1	0	0	0	1
	I_2	0	$P_{1A}^1(1 - P_{2A}^1)$	0	$1-\Sigma$
	I_3	0	$P_{1A}^1P_{2A}^2$	$1 - P_{2A}^1 - P_{2A}^2$	$1-\Sigma$
	I_∞	0	$P_{1A}^1P_{2A}^2$	P_{2A}^3	$1-\Sigma$
I_3	I_1	0	0	0	1
	I_2	0	$P_{1A}^1(1 - P_{2A}^1)$	0	$1-\Sigma$
	I_3	0	$P_{1A}^1P_{2A}^2$	$(P_{1A}^1 + P_{1A}^2)(1 - P_{2A}^1 - P_{2A}^2)$	$1-\Sigma$
	I_∞	0	$P_{1A}^1P_{2A}^2$	$(P_{1A}^1 + P_{1A}^2)P_{2A}^3$	$1-\Sigma$
I_∞	I_1	0	0	0	1
	I_2	0	$P_{1A}^1(1 - P_{2A}^1)$	0	$1-\Sigma$
	I_3	0	$P_{1A}^1P_{2A}^2$	$(P_{1A}^1 + P_{1A}^2)(1 - P_{2A}^1 - P_{2A}^2)$	$1-\Sigma$
	I_∞	0	$P_{1A}^1P_{2A}^2$	$(P_{1A}^1 + P_{1A}^2)P_{2A}^3$	$1-\Sigma$

•P_{ij}^k : The probability that an arc a_{ij} from node n_i to n_j is failed in the kth time interval

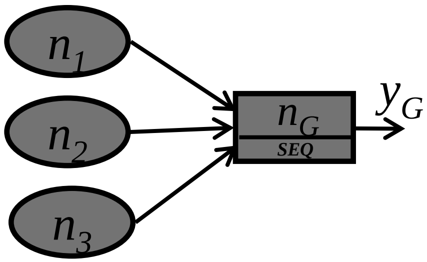
Probability Tables for Dynamic Nodes

- SPARE node



$f > x, y$	• 0
$f < x, y$	• $P(a_{1F}$ fails at the fth interval$)$ · $P(a_{2F}$ fails at or before the fth interval$)$ + $P(a_{1F}$ fails before the fth interval$)$ · $P(a_{2F}$ fails at the fth interval$)$
$x < f < y$	• $P(a_{2F}$ fails at the fth interval$)$
$y \leq f < x$	• $P(a_{1F}$ fails at the fth interval$)$
$f = x < y$	• $P(a_{1F}$ doesn't fail before the fth interval$)$ · $P(a_{2F}$ fails at or before the fth interval$)$ + $P(a_{1F}$ fails before the fth interval$)$ · $P(a_{2F}$ fails at the fth interval$)$
Else	• 1 -(sum of the other probabilities in the same row)

- SEQ node

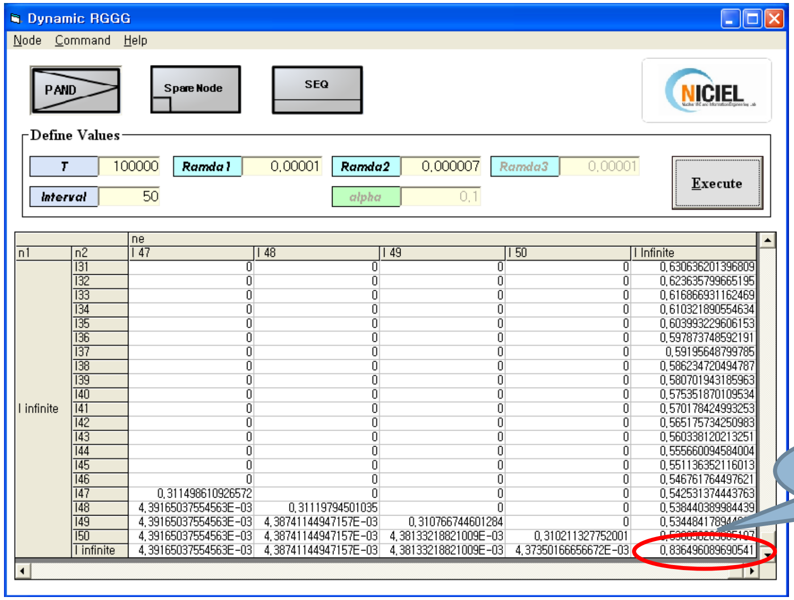


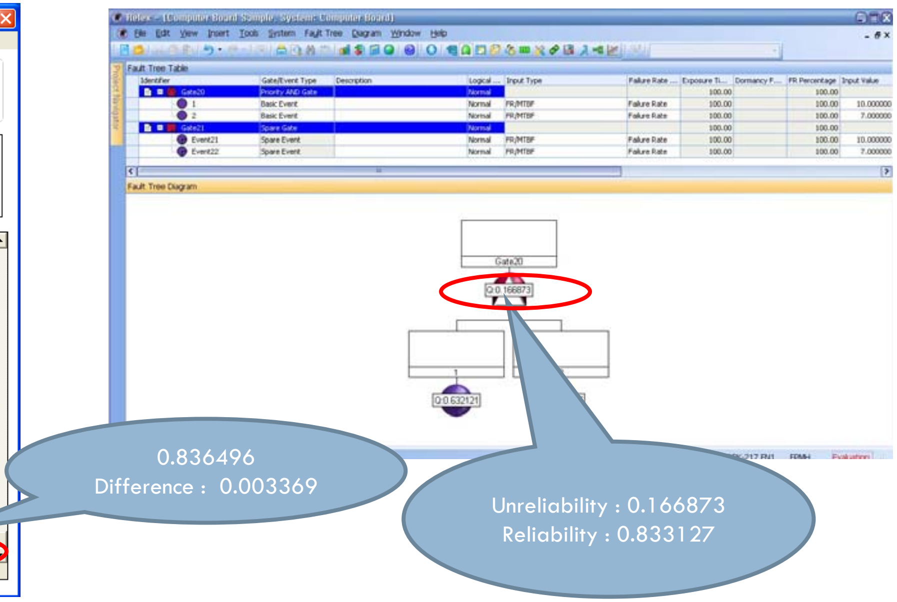
$g < 3$	• 0
$g \geq 3$	• $\sum$ $P(a_{1G}$ fails at the ath interval$)$ · $P(a_{2G}$ fails at the bth interval$)$ · $P(a_{3G}$ fails at the cth interval$)$ $(when\ a + b + c = g)$
$g = \infty$	• 1 – (sum of the other probabilities in the same row)

Development of a Tool for Generating PT

DRGGG

Relex Studio

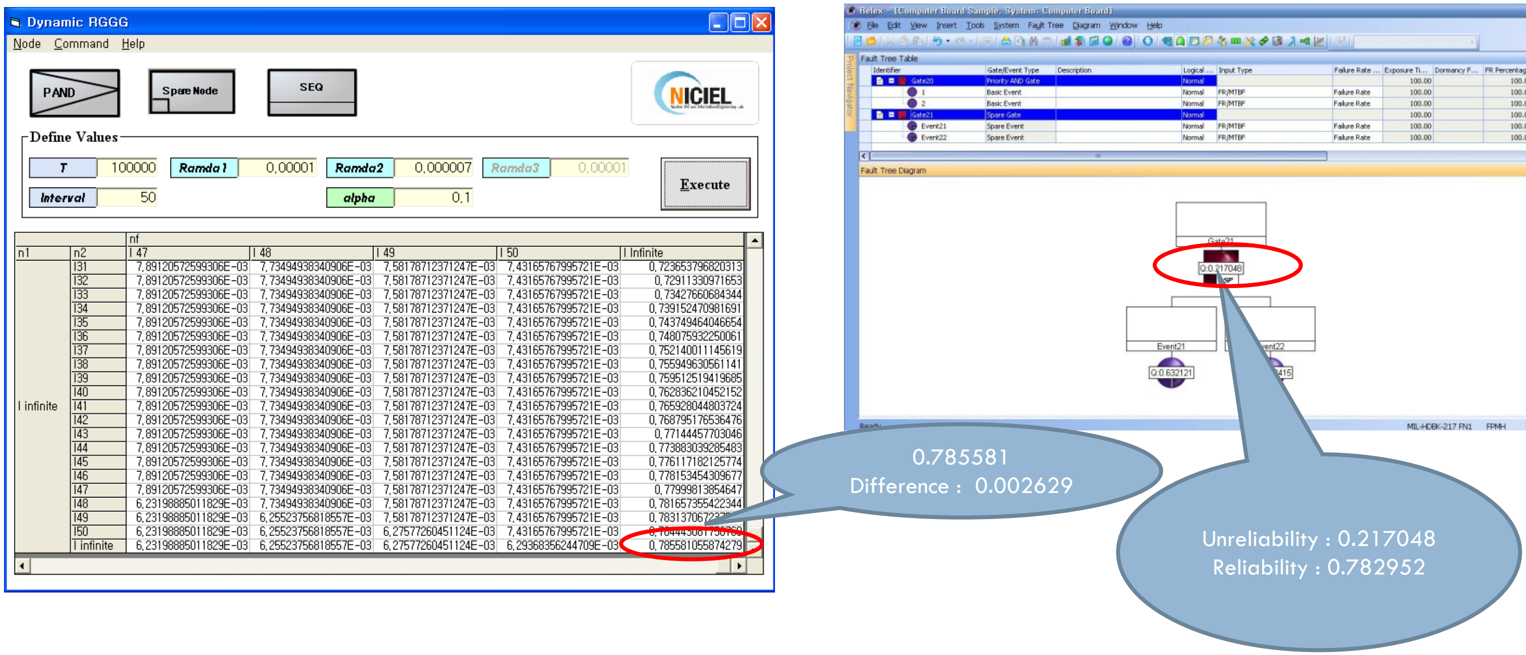




Development of a Tool for Generating PT

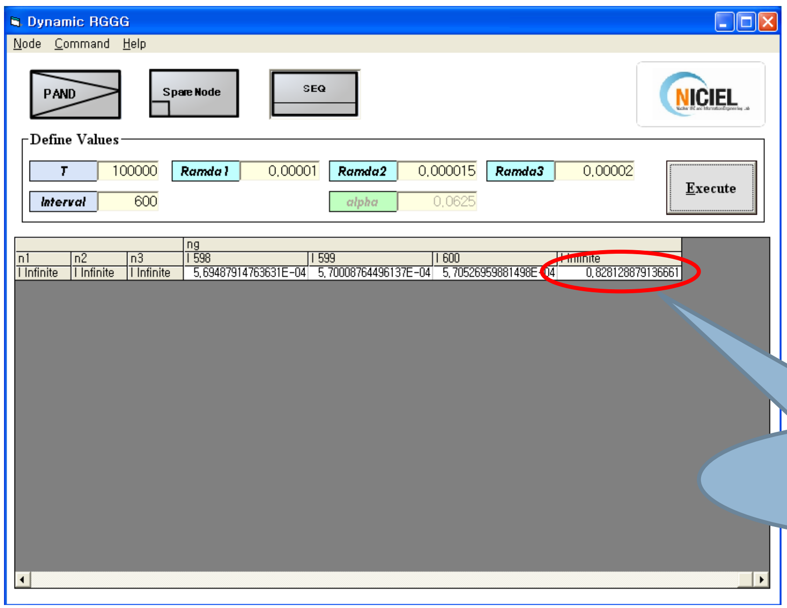
DRGGG

Relex Studio



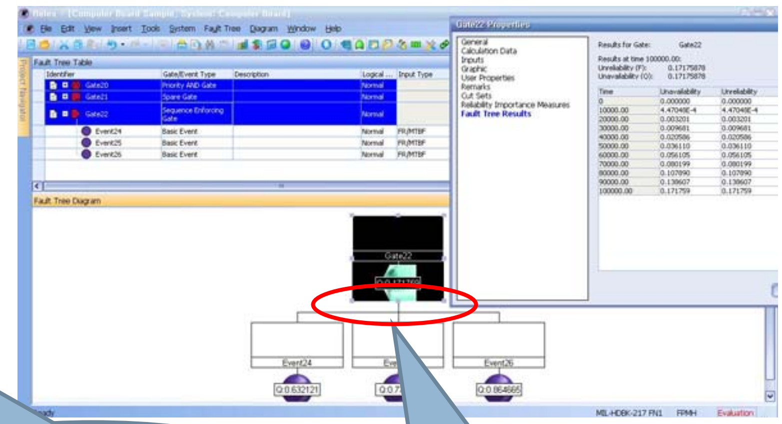
Development of a Tool for Generating PT

DRGGG



0.828129
Difference : 0.000112

Relex Studio



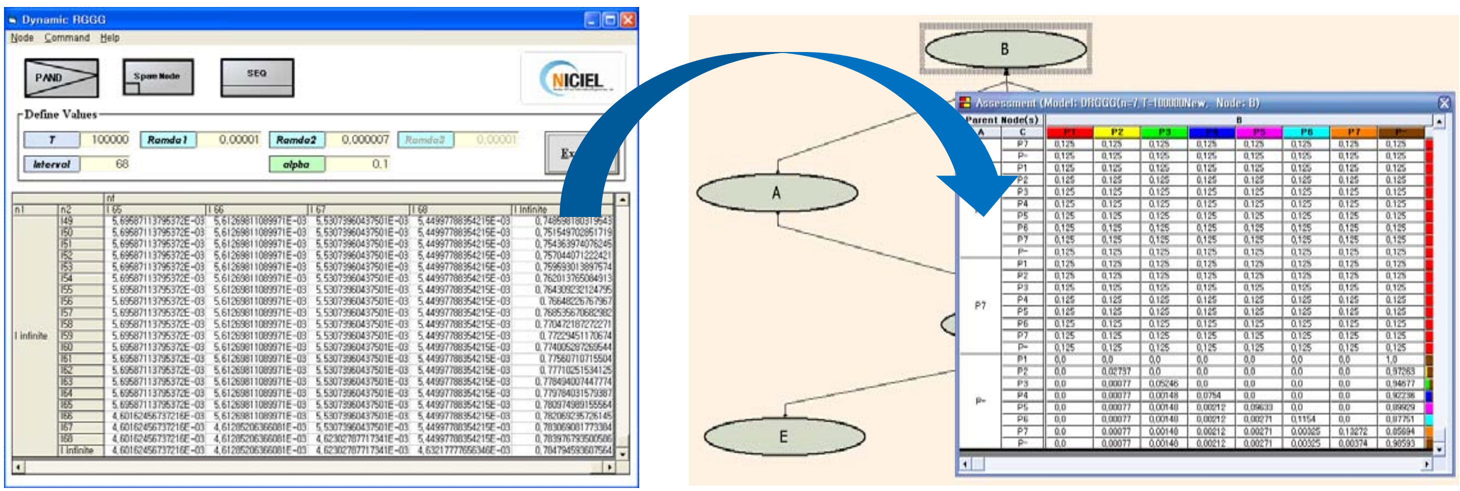
Unreliability : 0.171759
Reliability : 0.828241

Summary & Further Study

- We added dynamic nodes to RGGG in order to model a dynamic system intuitively.
- When analyzing reliability of a dynamic system through dynamic RGGG, making the probability tables is the most difficult work.
  - We adopted discret-time method to make probability tables.
  - The rule of filling probability tables was proposed.
  - As n(discretization number) increases, the result will be more accurate.

Summary & Further Study

- Further Study
  - Development of software tool
    - Reducing the calculation time.
    - Development of a tool for connecting the generated PT and a Bayesian Network tool.



References

- M. C. Kim and P. H. Seong, “Reliability graph with general gates: an intuitive and practical method for system reliability analysis”, Reliability Engineering and System Safety, 2002.
- J. B Dugan, S. J. Bavuso and M. A. Boyd, “Dynamic Fault-Tree Models for Fault-Tolerant Computer Systems”, IEEE Transactions on Reliability, 1992.
- H. Boudali, J. B. Dugan, “A discrete-time Bayesian network reliability modeling and analysis framework”, Reliability Engineering and System Safety, 2005.