



An Intuitive Method for Reliability Analysis of Dynamic Systems

KAIST



Seung Ki Shin, Gyoung Tae Goh and Poong Hyun Seong
Korea Advanced Institute of Science and Technology (KAIST)

2008 PSAM9
May 23rd, 2008

Contents

- *Introduction*
- *Reliability Graph with General Gates*
- *Adding Dynamic Nodes to RGGG*
- *Probability Tables for Dynamic Nodes*
 - *Discrete-time method*
- *Development of a Tool for Generating PT*
- *Summary & Further Study*
- *References*

Introduction

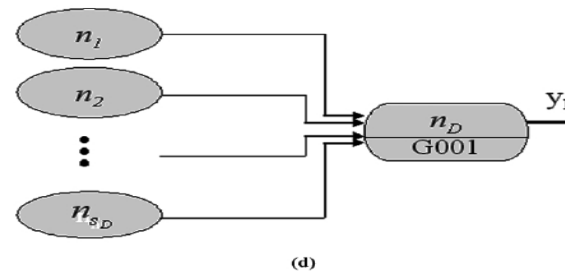
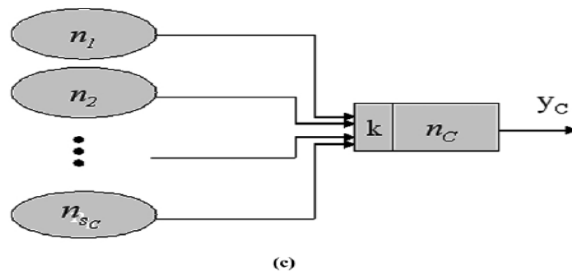
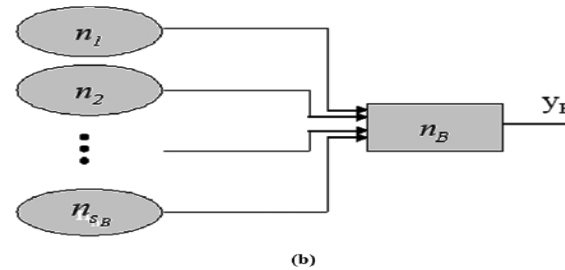
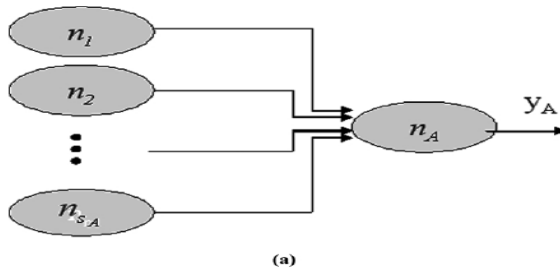
- Among the methods to analyze system reliability, fault tree analysis is the most widely used method, but it's not an intuitive method.
- Reliability graph with general gates (RGGG) method was proposed in order to overcome this shortcoming of fault tree.
- Conventional fault tree and RGGG method cannot capture the dynamic behavior of system associated with time dependent events.
- Dynamic fault tree method was proposed, but it's also not intuitive. And when calculating the reliability from dynamic fault tree, usually Markov chain method is used, but it has the state space explosion problem.
- If it is possible to add a sequential concept to RGGG, it will become an intuitive dynamic method.

Reliability Graph with General Graph

- Reliability graph is an intuitive method, so it can model systems by one-to-one match graph.
- The reason why it's not used widely is low expression power. (Only OR gate)
- RGGG which utilizes general gates was proposed.
- By determining the probability table for each node, RGGG can be transformed to an equivalent Bayesian network and the system reliability can be calculated.

Reliability Graph with General Graph

- RGGG nodes

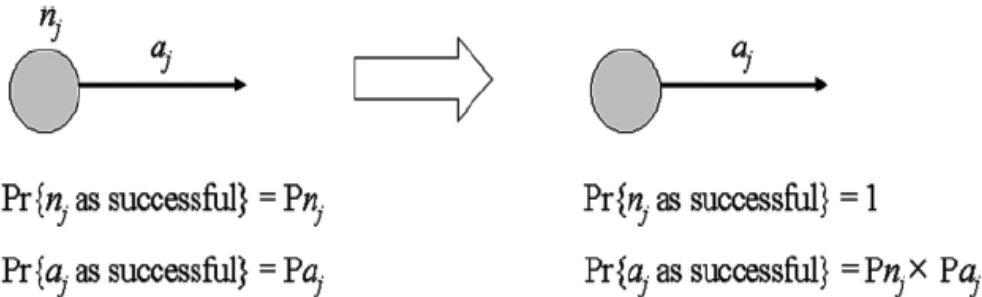


- (a) OR (b) AND (c) k-out-of-n (d) general purpose gate

Reliability Graph with General Graph

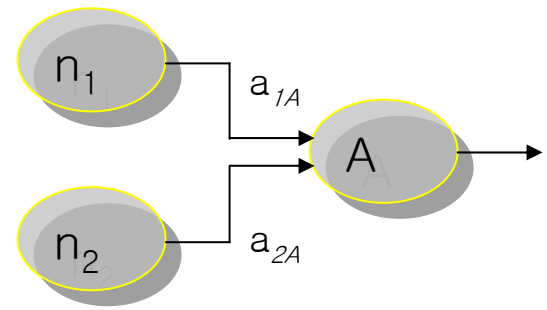


- Transformation to reliability graph with perfect node



Ex.) Probability table for a node with OR gate when $n = 2$

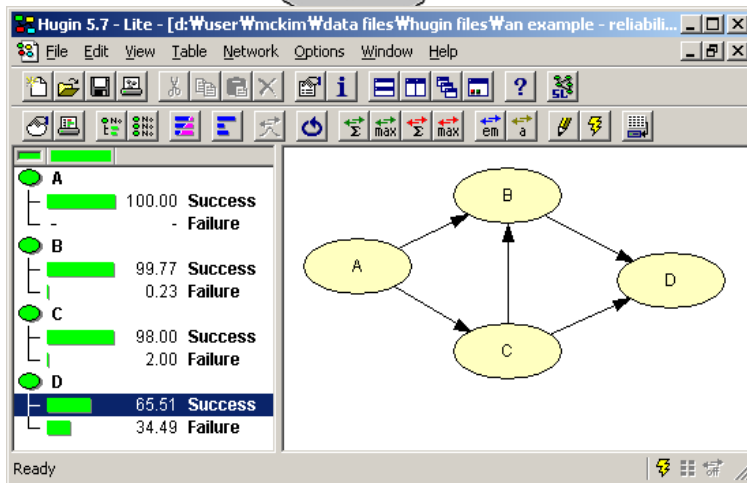
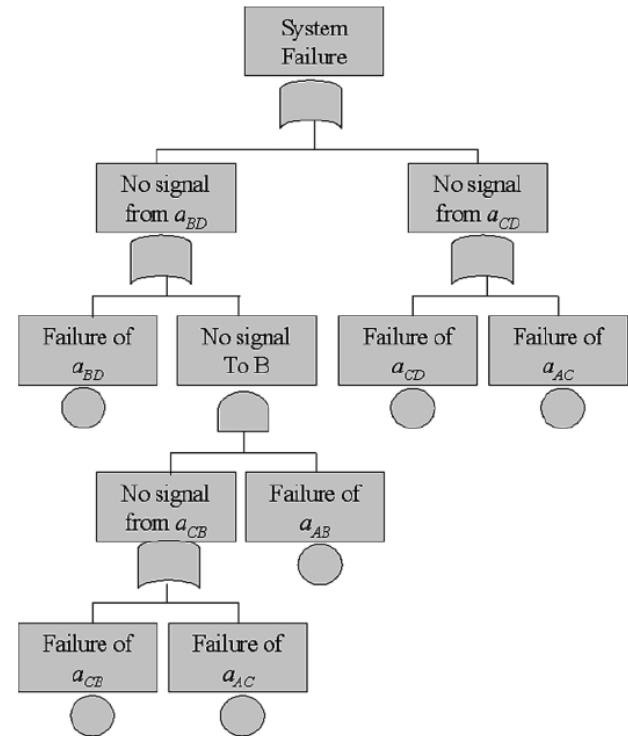
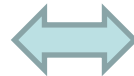
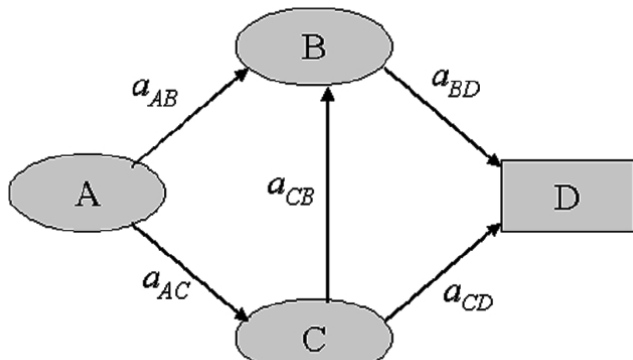
	$y_1 = 1$ (success)		$y_1 = 0$ (failure)	
	$y_2 = 1$ (success)	$y_2 = 0$ (failure)	$y_2 = 1$ (success)	$y_2 = 0$ (failure)
$y_A = 1$ (success)	$r_{1A} + r_{2A} - r_{1A}r_{2A}$	r_{1A}	r_{2A}	0
$y_A = 0$ (failure)	$1 - (r_{1A} + r_{2A} - r_{1A}r_{2A})$	$1 - r_{1A}$	$1 - r_{2A}$	1



Reliability Graph with General Graph

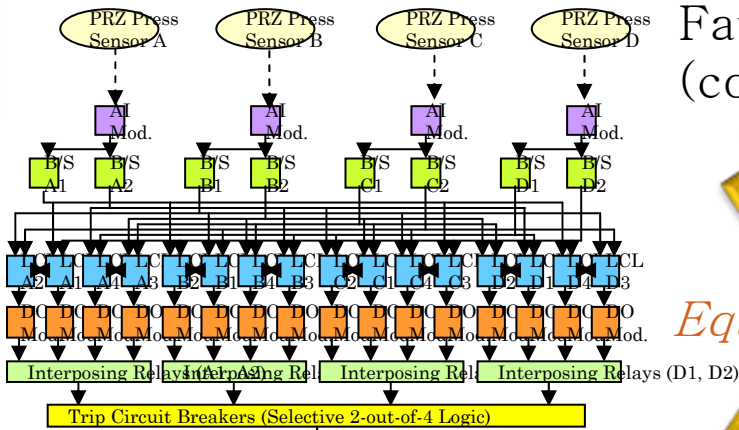


- Comparison of fault tree and RGGG

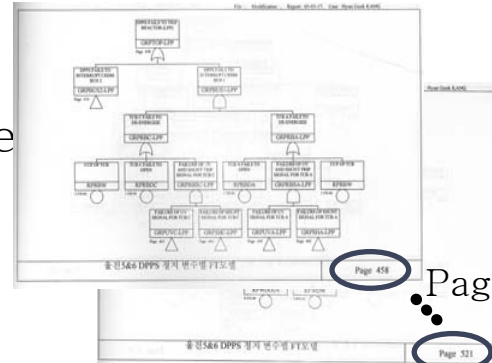


Reliability Graph with General Graph

Ex. 1) Digital Plant Protection System



Fault Tree (complex)



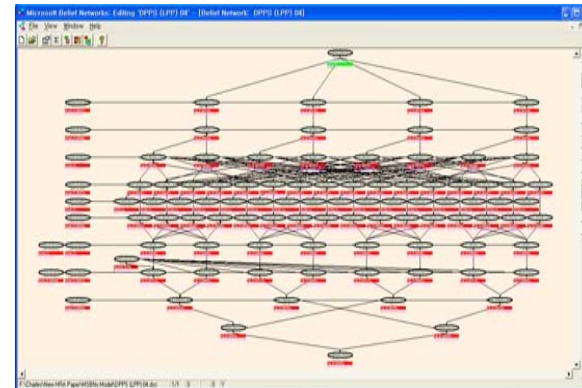
Page 458

Page 521

Equivalent !!!

64 pages !!!

Structure of DPPS
(For low pressurizer
pressure trip)

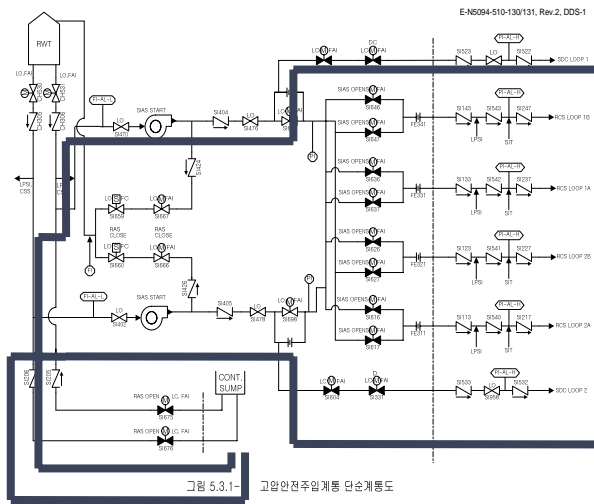


Only 1 page !!!

RGGG
(more intuitive)

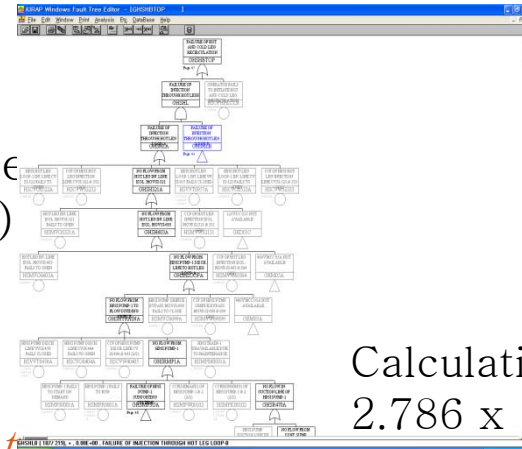
Reliability Graph with General Graph

Ex. 2) Recirculation in KSNPs



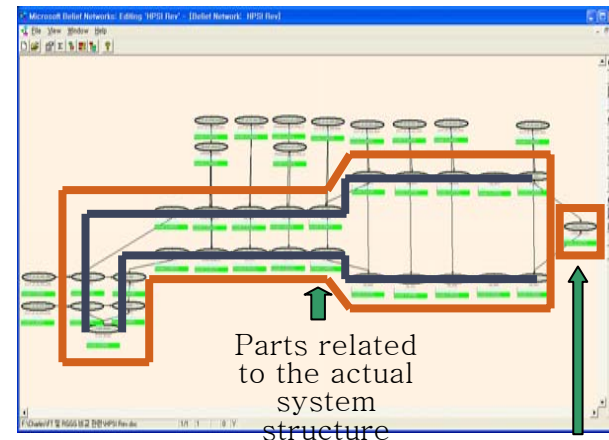
High Pressure Recirculation (in KSNP-type NPPs)

Fault Tree (complex)



Calculation result : 2.786×10^{-3}

Equivalent!!!



Calculation result : 2.7746×10^{-3}

RGGG (more intuitive)

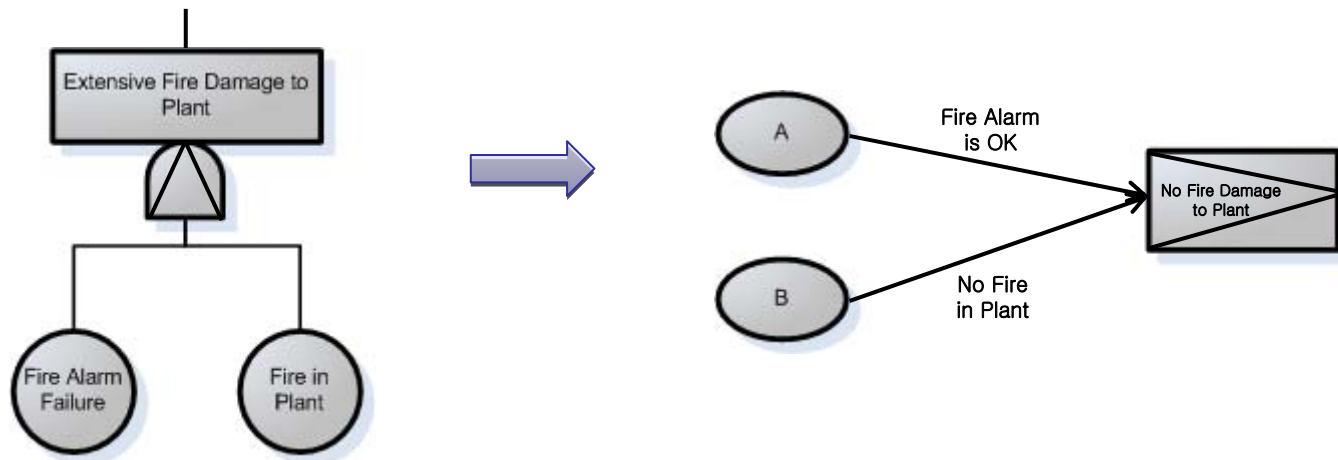
Adding Dynamic Nodes to RGGG

- The existing reliability graph with general gate (RGGG) method cannot capture dynamic behaviors of a system associated with sequence dependent failure events.
- We add dynamic nodes to existing RGGG and conduct quantitative analysis by using discrete-time method.
 - **Dynamic RGGG**

Adding Dynamic Nodes to RGGG

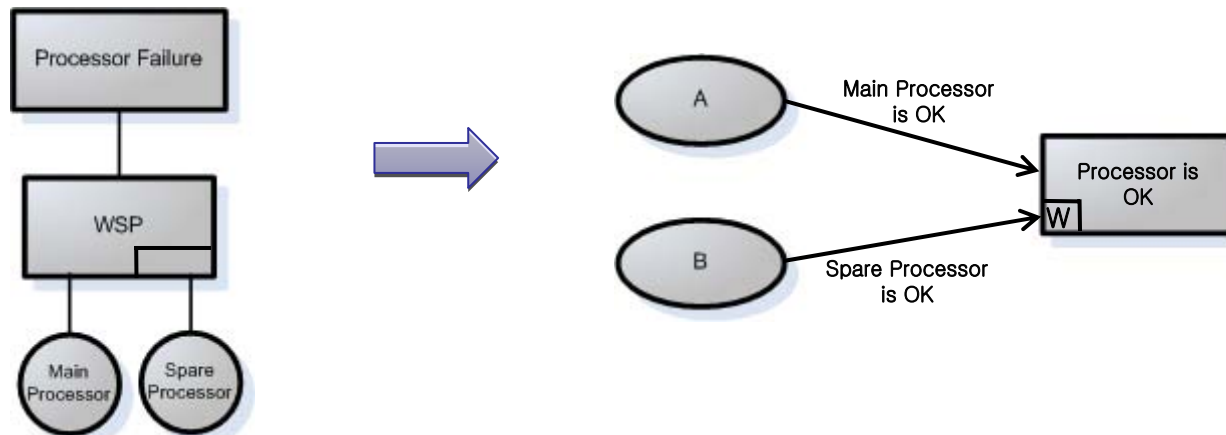
- Priority-AND (PAND) node

- Node fails only if both signals from A and B are disconnected and a signal from A is disconnected before B.



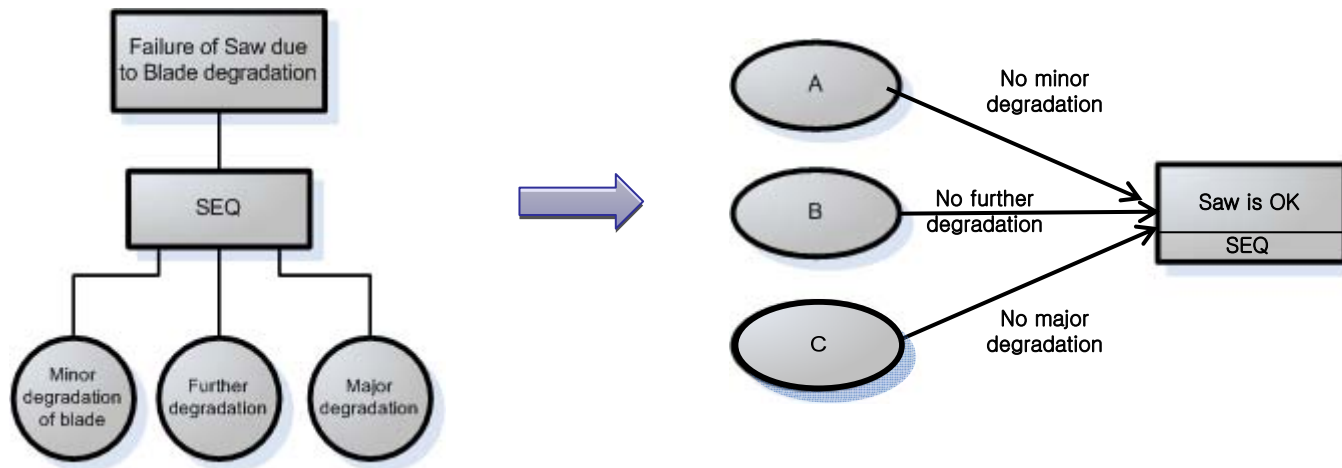
Adding Dynamic Nodes to RGGG

- Spare node
 - Node fails only if a primary signal and all spare signals are disconnected.



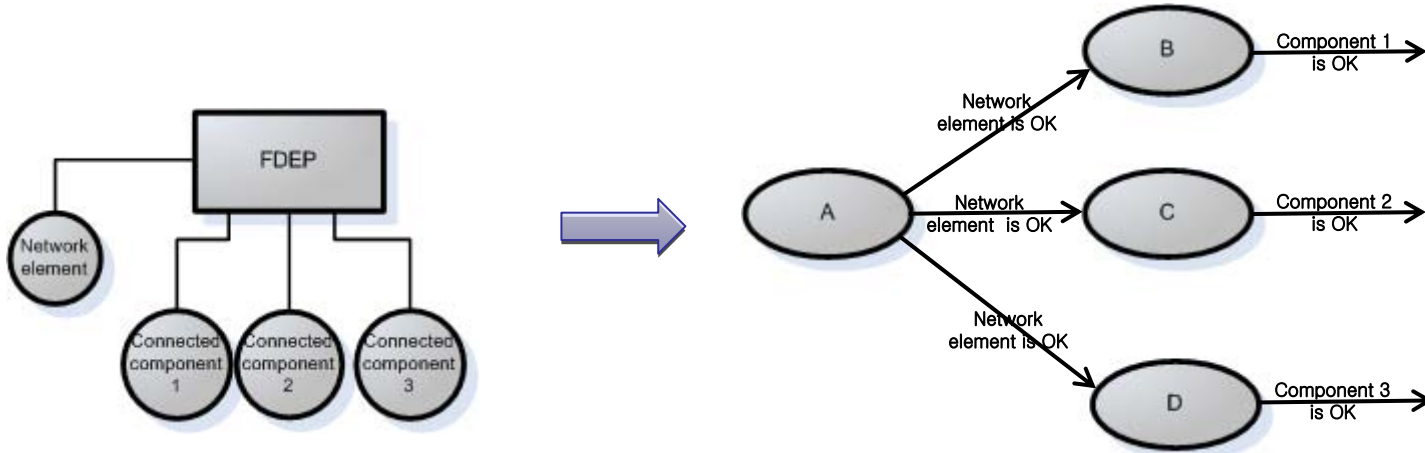
Adding Dynamic Nodes to RGGG

- SEQ node
 - The input signals are constrained to be disconnected in the particular order and SEQ node fails if and only if all input signals are disconnected.



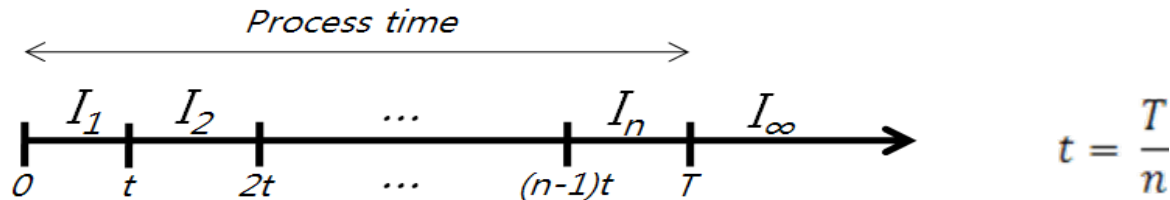
Adding Dynamic Nodes to RGGG

- FDEP node
 - This node is not necessary in RGGG.
 - Existing RGGG can illustrate the property of FDEP gate.



Probability Tables for Dynamic Nodes

- In order to transform a dynamic reliability graph to an equivalent Bayesian network, the probability table corresponding to each dynamic node should be determined.
 - The discrete-time method is used.



- Divide the process time line into n same intervals.
- Output of each node is one of $\{I_1, I_2, \dots, I_n, I_\infty\}$.
 - I_k means that the node is failed in k th interval.
 - I_∞ means that the node is never failed.
- P_{ij}^k : The probability that an arc a_{ij} from node n_i to n_j is failed in the k th time interval
 - When the cumulative failure distribution function of a_{ij} is $F_{ij}(t)$,

$$P_{ij}^k = \int_{(k-1)t}^{kt} \frac{dF_{ij}(t)}{dt} dt$$

Probability Tables for Dynamic Nodes

- In order to obtain the accurate reliability, n (the number of time discretization) should increase, but as n increases, the probability table of each node becomes more complex.

- The number of blanks that should be filled is $(n+1)^3$ per each node. (2 inputs)

Ex) $n = 3$,

		C			
A	B	I_1	I_2	I_3	I_∞
I_1	I_1				
	I_2				
	I_3				
	I_∞				
I_2	I_1				
	I_2				
	I_3				
	I_∞				
I_3	I_1				
	I_2				
	I_3				
	I_∞				
I_∞	I_1				
	I_2				
	I_3				
	I_∞				

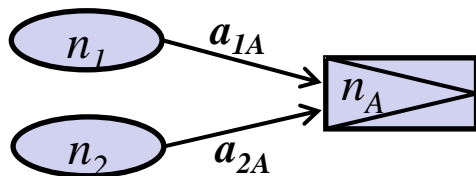
*64 blanks
should be
filled.*

- The rule of making probability tables is necessary to reduce making time and mistakes.

Probability Tables for Dynamic Nodes



■ PAND node



- Let output of n_1, n_2, n_A as I_x, I_y, I_z
 - I_x means that n_1 fails at x^{th} interval.

$y < z$	• 0
$x \geq z, y = z$	• $P(a_{1A}$ fails before z interval) • $(1 - P(a_{2A}$ fails before z interval))
$x \geq z, y > z$	• $P(a_{1A}$ fails before z interval) • $P(a_{2A}$ fails at z interval)
$x < z, y = z$	• $1 - P(a_{2A}$ fails before z interval)
$x < z, y > z$	• $P(a_{2A}$ fails at z interval)

Ex.) $n=3 \quad x, y, z \in \{1, 2, 3, \infty\}$

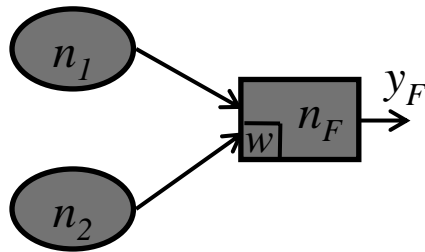
		n_A			
n_1	n_2	I_1	I_2	I_3	I_∞
I_1	I_1	0	0	0	1
	I_2	0	$1 - P_{2A}^1$	0	$1 - \Sigma$
	I_3	0	P_{2A}^2	$1 - P_{2A}^1 - P_{2A}^2$	$1 - \Sigma$
	I_∞	0	P_{2A}^2	P_{2A}^3	$1 - \Sigma$
I_2	I_1	0	0	0	1
	I_2	0	$P_{1A}^1(1 - P_{2A}^1)$	0	$1 - \Sigma$
	I_3	0	$P_{2A}^1 P_{1A}^1 P_{2A}^2$	$1 - P_{2A}^1 - P_{2A}^2$	$1 - \Sigma$
	I_∞	0	$P_{1A}^1 P_{2A}^2$	P_{2A}^3	$1 - \Sigma$
I_3	I_1	0	0	0	1
	I_2	0	$P_{1A}^1(1 - P_{2A}^1)$	0	$1 - \Sigma$
	I_3	0	$P_{1A}^1 P_{2A}^2$	$(P_{1A}^1 + P_{1A}^2)(1 - P_{2A}^1 - P_{2A}^2)$	$1 - \Sigma$
	I_∞	0	$P_{1A}^1 P_{2A}^2$	$P_{2A}^2(P_{1A}^1 + P_{1A}^2)P_{2A}^3$	$1 - \Sigma$
I_∞	I_1	0	0	0	1
	I_2	0	$P_{1A}^1(1 - P_{2A}^1)$	0	$1 - \Sigma$
	I_3	0	$P_{2A}^1 P_{1A}^1 P_{2A}^2$	$(P_{1A}^1 + P_{1A}^2)(1 - P_{2A}^1 - P_{2A}^2)$	$1 - \Sigma$
	I_∞	0	$P_{1A}^1 P_{2A}^2$	$(P_{1A}^1 + P_{1A}^2)P_{2A}^3$	$1 - \Sigma$

• P_{ij}^k : The probability that an arc a_{ij} from node n_i to n_j is failed in the k th time interval

Probability Tables for Dynamic Nodes

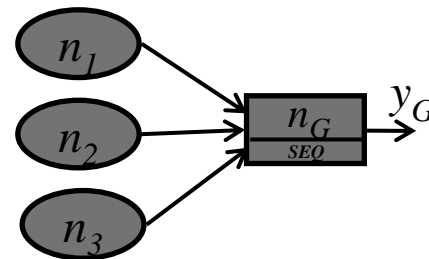


SPARE node



$f > x, y$	• 0
$f < x, y$	• $P(a_{1F}$ fails at the f th interval) · $P(a_{2F}$ fails at or before the f th interval) + $P(a_{1F}$ fails before the f th interval) · $P(a_{2F}$ fails at the f th interval)
$x < f < y$	• $P(a_{1F}$ fails at the f th interval)
$y \leq f < x$	• $P(a_{1F}$ fails at the f th interval)
$f = x < y$	• $P(a_{1F}$ doesn't fail before the f th interval) · $P(a_{2F}$ fails at or before the f th interval) + $P(a_{1F}$ fails before the f th interval) · $P(a_{2F}$ fails at the f th interval)
Else	• 1 - (sum of the other probabilities in the same row)

SEQ node



$g < 3$	• 0
$g \geq 3$	• $\sum P(a_{1G}$ fails at the a th interval) · $P(a_{2G}$ fails at the b th interval) · $P(a_{3G}$ fails at the c th interval) (when $a + b + c = g$)
$g = \infty$	• 1 - (sum of the other probabilities in the same row)

Development of a Tool for Generating PT



DRGGG

Dynamic RGGG

Node Command Help

PAWD Spare Node SEQ

Define Values

T 100000 Randa1 0.00001 Randa2 0.000007 Randa3 0.00001

Interval 50 alpha 0.1

Execute

n1	n2	ne	1.47	1.48	1.49	1.50	1 Infinite
131		0	0	0	0	0	0.630636201396809
132		0	0	0	0	0	0.623635799665195
133		0	0	0	0	0	0.616686991162469
134		0	0	0	0	0	0.61032188054634
135		0	0	0	0	0	0.603993229606153
136		0	0	0	0	0	0.597673748692191
137		0	0	0	0	0	0.591354208799785
138		0	0	0	0	0	0.585034720494787
139		0	0	0	0	0	0.5787151943185963
140		0	0	0	0	0	0.57239561870109534
141		0	0	0	0	0	0.566076094493253
142		0	0	0	0	0	0.5597565734250963
143		0	0	0	0	0	0.553437052013251
144		0	0	0	0	0	0.547117530609484004
145		0	0	0	0	0	0.5407980094584004
146		0	0	0	0	0	0.53447848817891447
147		0.311498610826572	0	0	0	0	0.5281589673665447
148		4.39165037554563E-03	0.31119794501035	0	0	0	0.5218394464584439
149		4.39165037554563E-03	4.38741144947157E-03	0.310766744601284	0	0	0.5155200000000000
150		4.39165037554563E-03	4.38741144947157E-03	4.38133218821009E-03	0.310211327752001	0	0.5092005000000000
1 Infinite		4.39165037554563E-03	4.38741144947157E-03	4.38133218821009E-03	4.3795016665672E-03	0.836496	0.836496009690541

Relex Studio

Relex - [Computer Board Sample - system - Computer Board]

Fault Tree Table

Identifier	Gate/Event Type	Description	Logical	Input Type	Failure Rate	Exposure T...	Concomity P...	PR Percentage	Input Value
Gate3	Priority AND Gate		Normal			100.00	100.00	100.00	
1	Basic Event		Normal	PR/PTSP	Failure Rate	100.00	100.00	10.00000	
2	Basic Event		Normal	PR/PTSP	Failure Rate	100.00	100.00	7.00000	
Gate21	Spare Gate		Normal			100.00	100.00	100.00	
Event21	Spare Event		Normal	PR/PTSP	Failure Rate	100.00	100.00	10.00000	
Event22	Spare Event		Normal	PR/PTSP	Failure Rate	100.00	100.00	7.00000	

Fault Tree Diagram

0.836496
Difference : 0.003369

Unreliability : 0.166873
Reliability : 0.833127

Development of a Tool for Generating PT



DRGGG

Relex Studio

Dynamic RGGG

Node Command Help

PAND Spare Node SEQ

Define Values

T 100000 Ramda1 0.00001 Ramda2 0.000007 Ramda3 0.00001

Interval 50 alpha 0.1

Execute

n1	n2	nf	I 47	I 48	I 49	I 50	I Infinite
131	7.8912057259930E-03	7.73494938340906E-03	7.58178712371247E-03	7.43165767995721E-03	0.723653796820313		
132	7.8912057259930E-03	7.73494938340906E-03	7.58178712371247E-03	7.43165767995721E-03	0.72911330971653		
133	7.8912057259930E-03	7.73494938340906E-03	7.58178712371247E-03	7.43165767995721E-03	0.73427660684344		
134	7.8912057259930E-03	7.73494938340906E-03	7.58178712371247E-03	7.43165767995721E-03	0.739152470981691		
135	7.8912057259930E-03	7.73494938340906E-03	7.58178712371247E-03	7.43165767995721E-03	0.743749464046654		
136	7.8912057259930E-03	7.73494938340906E-03	7.58178712371247E-03	7.43165767995721E-03	0.74897532250061		
137	7.8912057259930E-03	7.73494938340906E-03	7.58178712371247E-03	7.43165767995721E-03	0.752140011145619		
138	7.8912057259930E-03	7.73494938340906E-03	7.58178712371247E-03	7.43165767995721E-03	0.755949630561141		
139	7.8912057259930E-03	7.73494938340906E-03	7.58178712371247E-03	7.43165767995721E-03	0.759512519419685		
140	7.8912057259930E-03	7.73494938340906E-03	7.58178712371247E-03	7.43165767995721E-03	0.762836210452152		
141	7.8912057259930E-03	7.73494938340906E-03	7.58178712371247E-03	7.43165767995721E-03	0.765928044903724		
142	7.8912057259930E-03	7.73494938340906E-03	7.58178712371247E-03	7.43165767995721E-03	0.768795176536476		
143	7.8912057259930E-03	7.73494938340906E-03	7.58178712371247E-03	7.43165767995721E-03	0.77144457703046		
144	7.8912057259930E-03	7.73494938340906E-03	7.58178712371247E-03	7.43165767995721E-03	0.773863039265403		
145	7.8912057259930E-03	7.73494938340906E-03	7.58178712371247E-03	7.43165767995721E-03	0.776117182125774		
146	7.8912057259930E-03	7.73494938340906E-03	7.58178712371247E-03	7.43165767995721E-03	0.778153454309577		
147	7.8912057259930E-03	7.73494938340906E-03	7.58178712371247E-03	7.43165767995721E-03	0.77999813854647		
148	6.23198885011829E-03	6.25523756818557E-03	7.58178712371247E-03	7.43165767995721E-03	0.781857355422344		
149	6.23198885011829E-03	6.25523756818557E-03	7.58178712371247E-03	7.43165767995721E-03	0.783137067223366		
150	6.23198885011829E-03	6.25523756818557E-03	6.2757260451124E-03	7.43165767995721E-03	0.784443061730766		
T Infinite	6.23198885011829E-03	6.25523756818557E-03	6.2757260451124E-03	6.29368356244709E-03	0.785581055874279		

Relex - [Computer Board Sample - System: Computer Board]

File Edit View Insert Tools System Fault Tree Diagram Window Help

Identifier	Gate/Event Type	Description	Logical	Input Type	Failure Rate	Exposure T...	Dormancy F...	FR Percentage	Input Value
1	Basic Event	Normal	Normal	FR/NTBF	100.00	100.00	100.00	10.00000	10.00000
2	Basic Event	Normal	Normal	FR/NTBF	100.00	100.00	100.00	7.00000	7.00000
Event21	Spare Event	Normal	Normal	FR/NTBF	100.00	100.00	100.00	10.00000	10.00000
Event22	Spare Event	Normal	Normal	FR/NTBF	100.00	100.00	100.00	7.00000	7.00000

Fault Tree Diagram

0.785581
Difference : 0.002629

Unreliability : 0.217048
Reliability : 0.782952

Development of a Tool for Generating PT



DRGGG

Relax Studio

Dynamic RGGG

Node Command Help

Define Values

T: 100000, Ramda1: 0.00001, Ramda2: 0.000015, Ramda3: 0.00002, Interval: 600, alpha: 0.0625

n1	n2	n3	n4	n5	n6	n7
Infinite	Infinite	Infinite	5.69487914763631E-04	5.70008764496137E-04	5.70526959681498E-04	0.828128879136561

Execute

Relax Studio

Results for Gate: Gate22

Time	Unreliability	Reliability
0	0.00000	0.00000
10000.00	4.47048E-4	4.47048E-4
20000.00	0.00101	0.00101
30000.00	0.00161	0.00161
40000.00	0.00206	0.00206
50000.00	0.00241	0.00241
60000.00	0.00269	0.00269
70000.00	0.00289	0.00289
80000.00	0.00299	0.00299
90000.00	0.10790	0.10790
100000.00	0.13967	0.13967
1000000.00	0.171759	0.171759

0.828129
Difference : 0.000112

Unreliability : 0.171759
Reliability : 0.828241

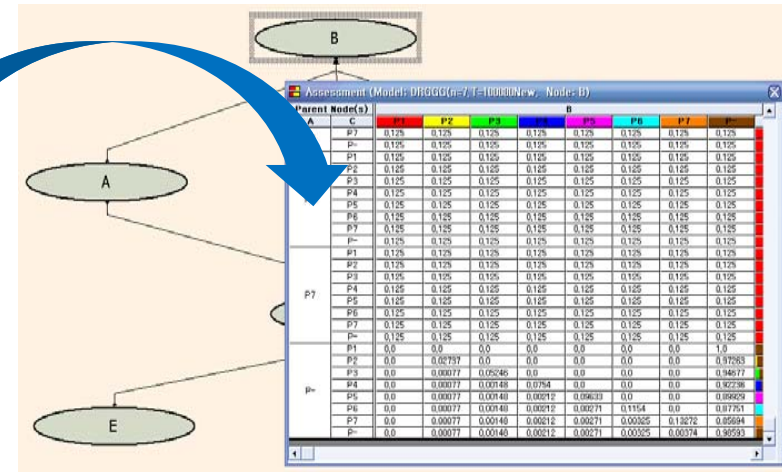
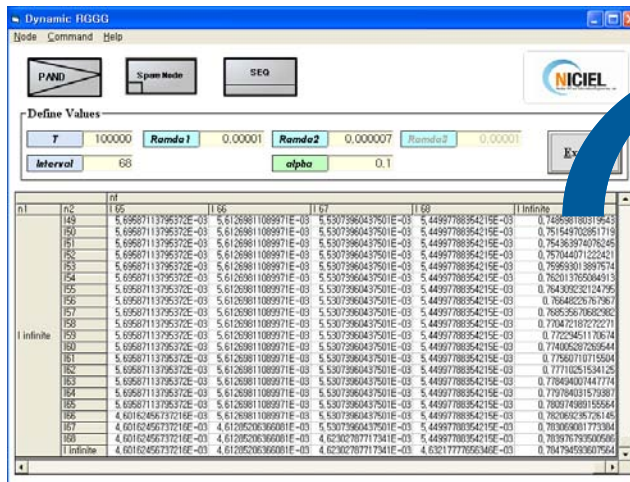
Summary & Further Study

- We added dynamic nodes to RGGG in order to model a dynamic system intuitively.
- When analyzing reliability of a dynamic system through dynamic RGGG, making the probability tables is the most difficult work.
 - We adopted discret-time method to make probability tables.
 - The rule of filling probability tables was proposed.
 - As n (discretization number) increases, the result will be more accurate.

Summary & Further Study



- Further Study
 - Development of software tool
 - Reducing the calculation time.
 - Development of a tool for connecting the generated PT and a Bayesian Network tool.



References

- M. C. Kim and P. H. Seong, “Reliability graph with general gates: an intuitive and practical method for system reliability analysis”, Reliability Engineering and System Safety, 2002.
- J. B Dugan, S. J. Bavuso and M. A. Boyd, “Dynamic Fault-Tree Models for Fault-Tolerant Computer Systems”, IEEE Transactions on Reliability, 1992.
- H. Boudali, J. B. Dugan, “A discrete-time Bayesian network reliability modeling and analysis framework”, Reliability Engineering and System Safety, 2005.