Vulnerability analysis of a power transmission system

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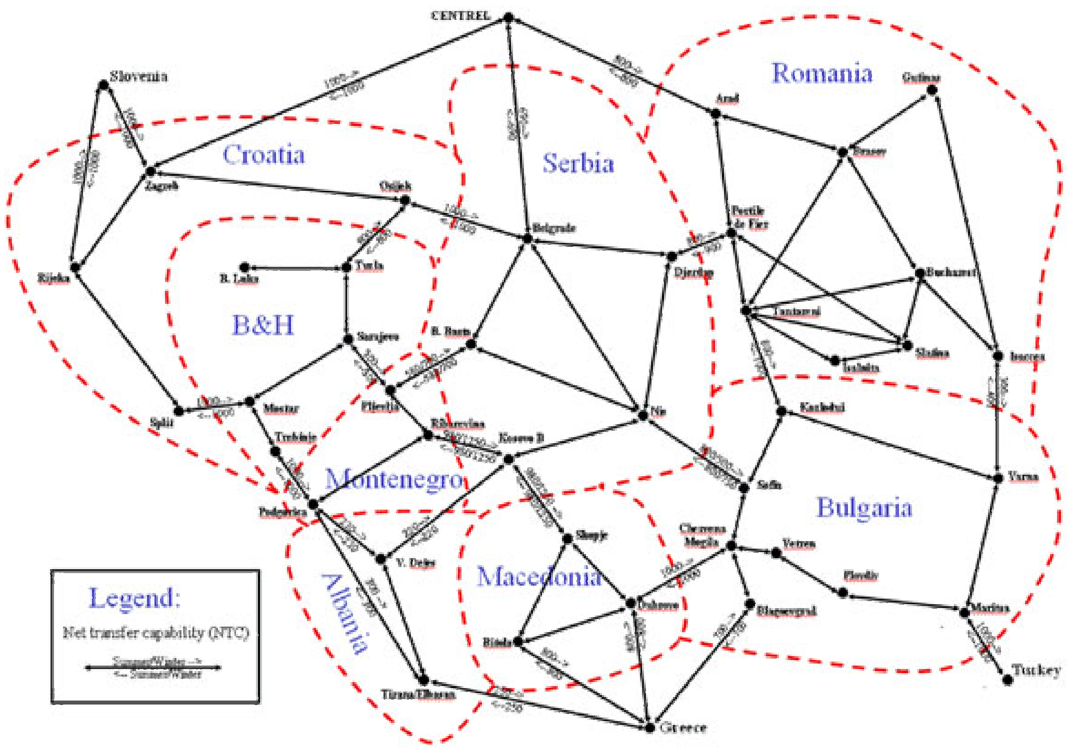
The System

Network infrastructures for electric power transmission



COMPLEXITY

The problem

Network infrastructures for electric power transmission

robustness (random failures)   resilience (attacks)

Methodologies

Robustness to failure?

Vulnerability to attacks?

Detailed simulation

PRA Tools (ET/FT)

Average indicators Modeling

Contribution of this work

Average indicators modeling:

- topological analysis
- reliability-weighted topological analysis

o validation by Monte Carlo simulation

Topological model

- Network infrastructure = Connected graph, **G**=(N,K)

- Adjacency matrix $\{a_{ij}\}$: a_{ij}=1 if there is an edge joining node i to node j and 0 otherwise

- d_{ij} = shortest path length from node i to node j

Topological indicators

- Global efficiency = measure of how good the nodes communicate through the network

$$E = \frac{1}{N(N-1)} \sum_{i,j \in N,\, i \neq j} \frac{1}{d_{ij}}$$

- Local efficiency = measure of the connectivity of the subgraph of the neighbors of a generic node *i*

$$E_{loc} = \frac{1}{N} \sum_{i \in G} E(G_i), \; where \; E(G_i) = \frac{1}{k_i(k_i - 1)} \sum_{l \neq m \in G_i} \frac{1}{d_{lm}}$$

Reliability-weighted topological model

- p_{ij} = connection reliability = probability that the transmission between nodes *i* and *j* occurs by the requirements
- Reliability matrix $\{p_{ij}\}$
- Most reliable path "length" from node *i* to node *j* :

$$d_{ij} = \min_{\gamma_{ij}} \left(\frac{1}{\prod_{mn \in \gamma_{ij}} p_{mn}} \right) \qquad 1 \le d_{ij} \le \infty$$

Reliability indicators

- Global reliability efficiency = measures the network connection characteristics on a global scale, accounting for the reliability of the edges in providing the power transmission

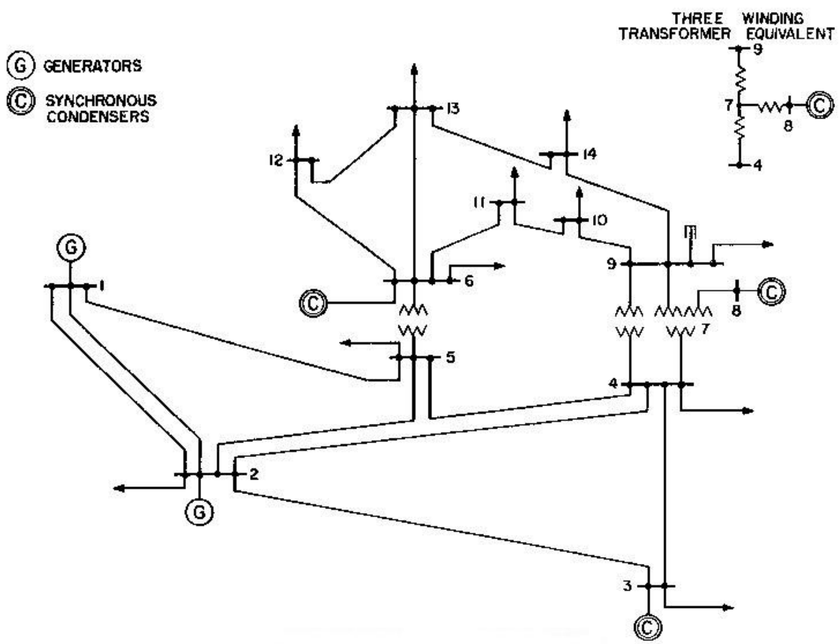
$$E_r = \frac{1}{N(N-1)} \sum_{i,j \in N,\, i \neq j} \frac{1}{d_{ij}}$$

- Local reliability efficiency = measures how much the network is fault tolerant in that it shows how reliable the power transmission remains among the first neighbours of *i* when *i* is removed

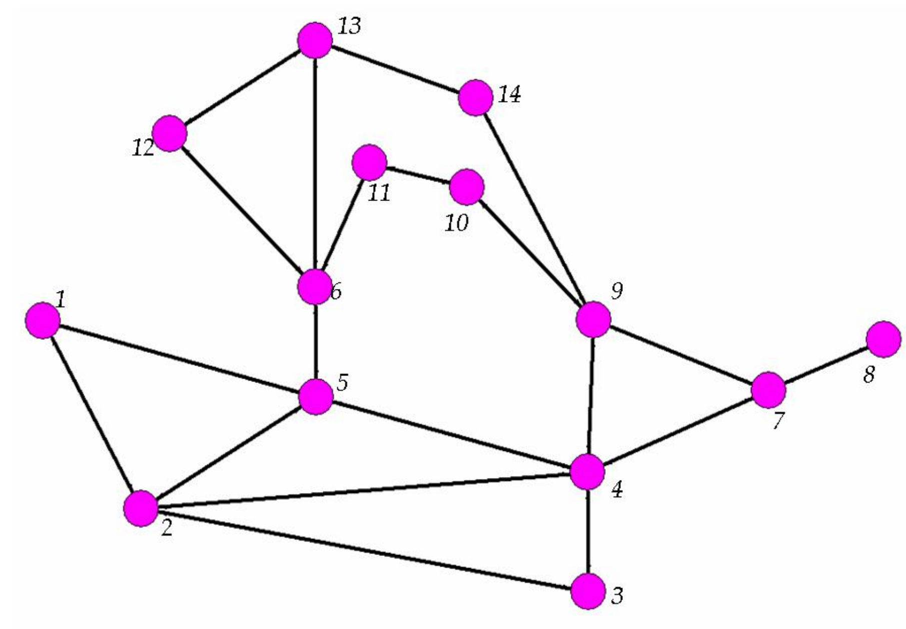
$$E_{rloc} = \frac{1}{N} \sum_{i=1 \in G} E_r(G_i),\ where\ E_r(G_i) = \frac{1}{k_i(k_i - 1)} \sum_{l \neq m \in G_i} \frac{1}{d_{lm}}$$

Case study: IEEE 14 BUS (American Electrical Power System)

Physical system



Network graph G(14,20)



Topological analysis: results

<u>IEEE 14 BUS Network</u>

$D = 5.000;\quad \overline{K} = 2.857$

$L = 2.374;\quad C = 0.367$

$E = 0.522;\quad E_{loc} = 0.392$

<u>Random Network</u>

$D = \infty;\quad \overline{K} = 2.428$

$L = \infty;\quad C = 0.167$

$E = 0.403;\quad E_{loc} = 0.167$

IEEE 14 BUS: values of global and local efficiencies larger than the random network

EEE 14 BUS ~ small-world network (good robustness propertie

Reliability analysis vs. Monte Carlo validation

IEEE 14 BUS Network

$$E_r = 0.3104$$

$$E_{rloc} = 0.1864$$

$$T_{CPU} = 1 \text{ s}$$

Monte Carlo simulation (N_{MC}=100000)

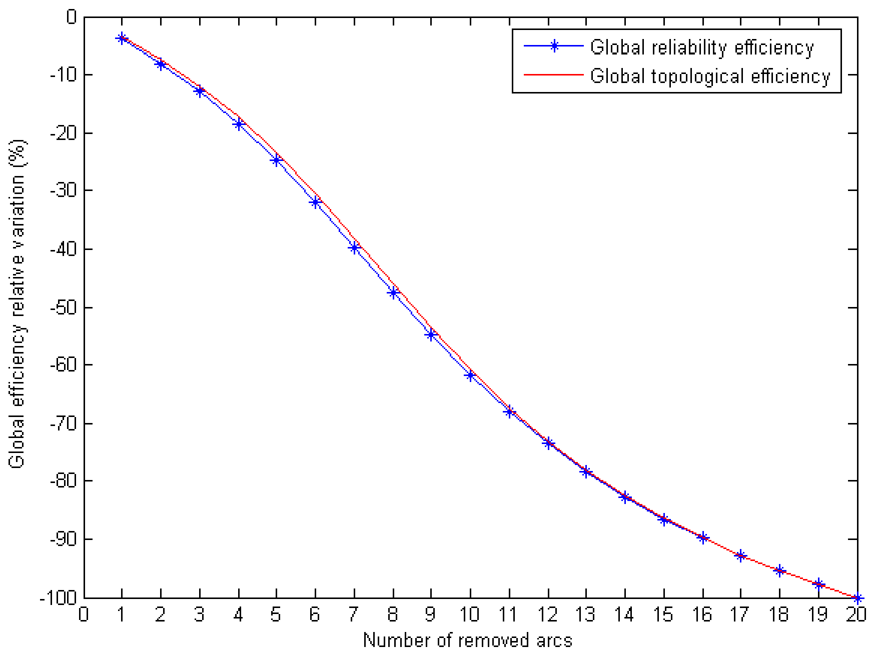
$$E_{MC} = 0.2801 \pm 0.0010$$

$$E_{MCloc} = 0.1434 \pm 0.0014$$

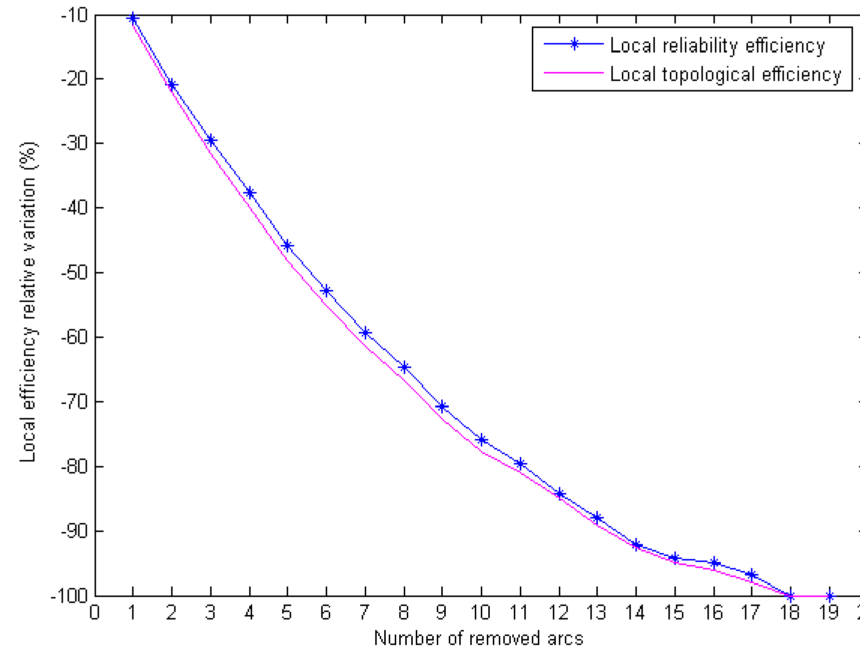
$$T_{CPU} = 33 \text{ s}$$

Topological and reliability robustness analysis

Random removal of a progressive number of arcs



Relative variation of global topological efficiency ΔE and global reliability efficiency ΔE_r



Relative variation of local topological efficiency ΔE_{loc} and local reliability efficiency ΔE_{rloc}

Topological and reliability resilience analysis

Removal of one node at a time (and of all the arcs incident onto it):

•Network nodes ranking according to the relative variation of topological and r

Rank	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$\Delta E/E$	4	7	9	6	5	2	13	14	10	11	3	1	12	8
$\Delta E_r/E_r$	7	9	4	6	5	13	14	10	11	8	12	2	3	1

•Network nodes ranking according to the relative variation of topological and reliability <u>local</u> efficiency caused by their removal

Rank	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$\Delta E_{loc}/E_{loc}$	2	4	5	12,13		6	1	3	9	7	10,11		8	14
$\Delta E_{rloc}/E_{rloc}$	4	2	12,13		6	9	5	7	1	3	11	10	14	8

Conclusions

Power transmission robustness and resilience analysis

↓ Computational burden

Topological and reliability-topological graph analysis

↓

Monte Carlo validation