Unavailability of a Redundant System with One Repair Team

Koichi Suyama and **Nobuko Kosugi**

Tokyo University of Marine Science and Technology

Outline

1. Introduction
2. Redundant system with one repair team
3. Markov analysis and its problem
4. Analysis based on scenario including awaiting repair
5. Monte Carlo simulations
6. Conclusion

1. Introduction

- **Redundant system (ex. m-out-of-n system)**
- **One repair team**

Until now

Markov analysis $\longrightarrow$ **availability, unavailability, ...**

⬇ **Problem?**

This paper

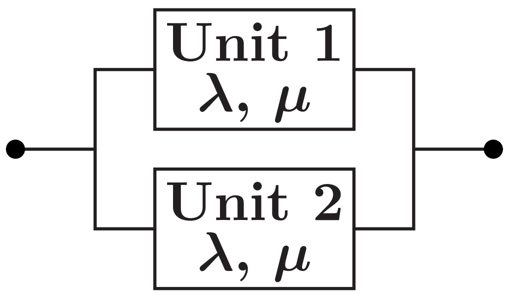
- **problem of Markov analysis**
- **new analysis based on mean awaiting-repair time**
  $\longrightarrow$ **new unavailability formula**
- **validity confirmation by Monte Carlo simulations**

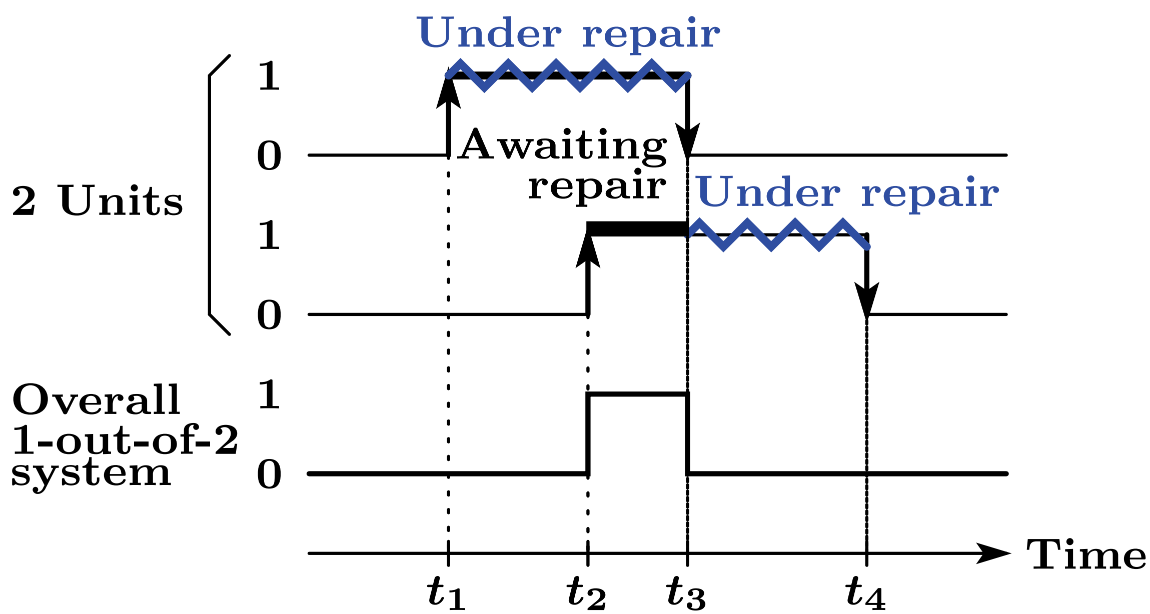
⬇

One solution to the problem

2. Redundant system with one repair team

1-out-of-2 system with identical units





failure: detected immediately

⬇ one repair team

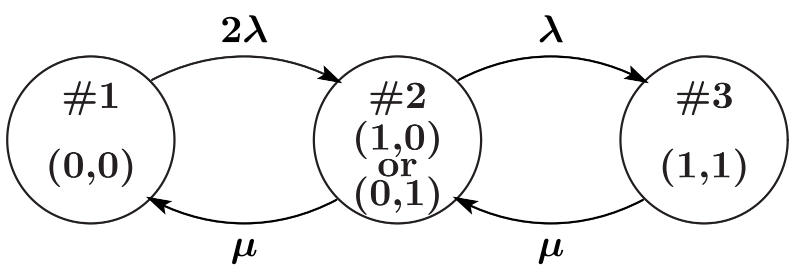
(i) another unit : normal (t_1) $\longrightarrow$ repair immediately starts

(ii) another unit : under repair (t_2)

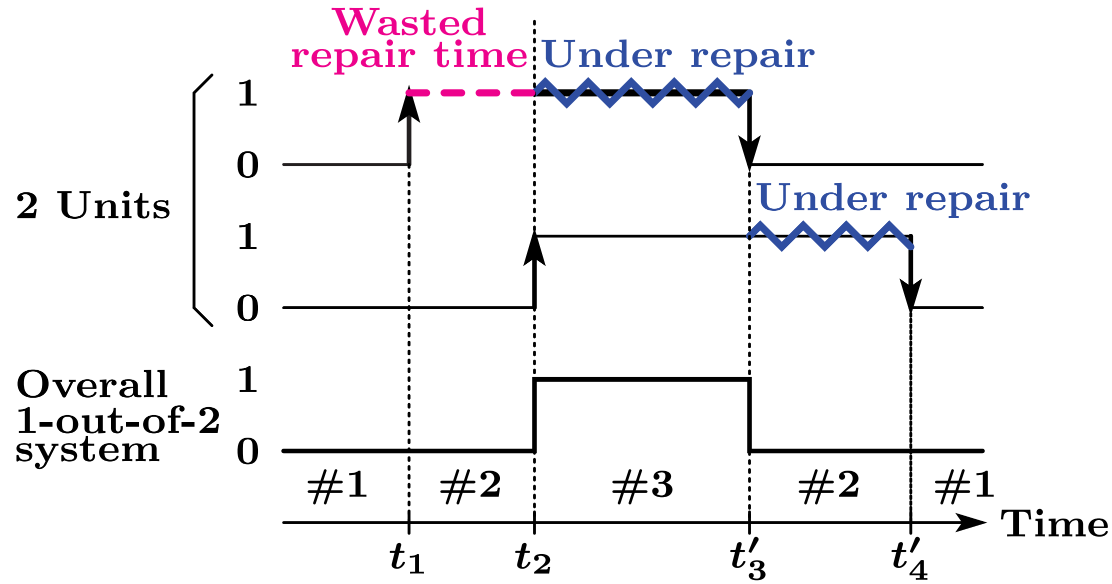
$\longrightarrow$ repair for 1st failure continues

newly failed unit : under awaiting repair

3. Markov analysis and its problem (1/2)



$$U_{\text{Markov}} = \frac{2\lambda^2}{2\lambda^2 + 2\lambda\mu + \mu^2}$$



Memory-less property

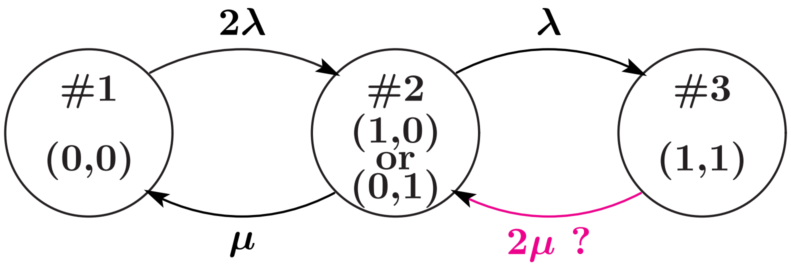
- state transition diagram
- exponential distribution

another failure in #2 ⟶ repair restarts after transition to #3

⬇

- stay time in #2 : wasted repair time
- unavalability $U_{\text{Markov}} >$ true

Measure against problem in IEC 61165



$$U'_{\mathrm{Markov}} = \frac{\lambda^2}{\lambda^2 + 2\lambda\mu + \mu^2} \ ?$$

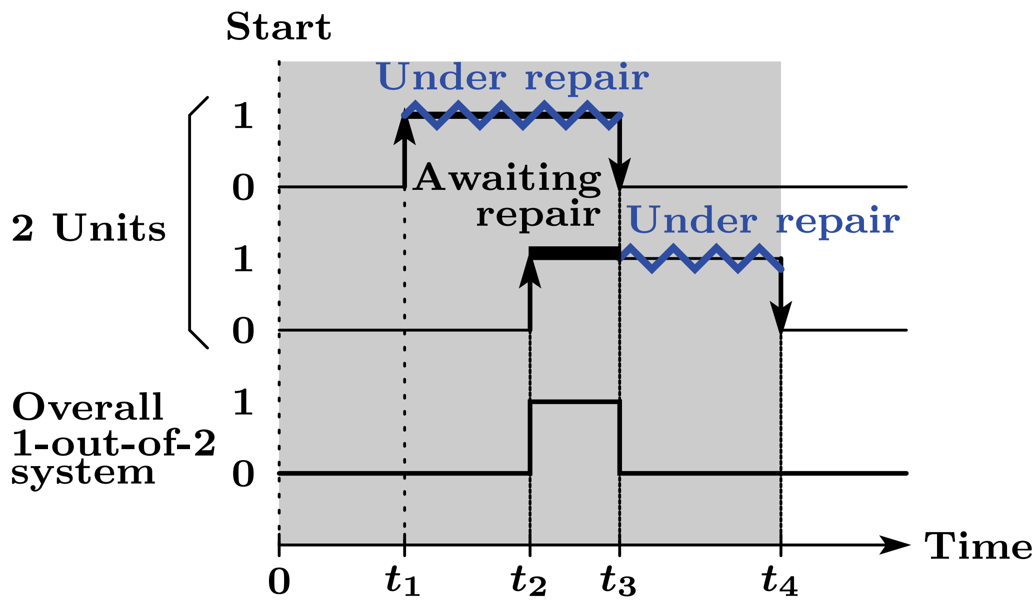
$\not\approx$ Monte Carlo simulation results

⬇

Problem unsolved !

4. Analysis based on scenario including awaiting repair (1/6)

Scenario $\longrightarrow$ **overcome memory-less property**



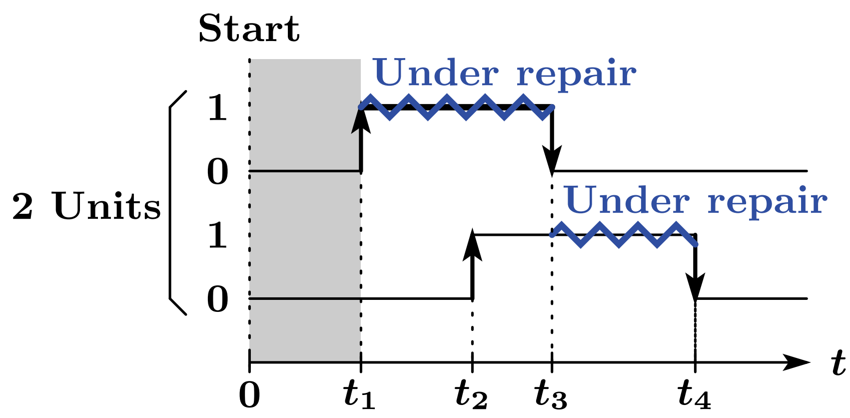
> [Approximation condition] Before repair starting after awaiting repair finishes, a new failure does not occur in another unit.

1-out-of-2 system falls into down only when 2nd failure occurs during repair for a unit which fails from Normal

⬇ No problem if $\frac{\lambda}{\mu} < 0.2$

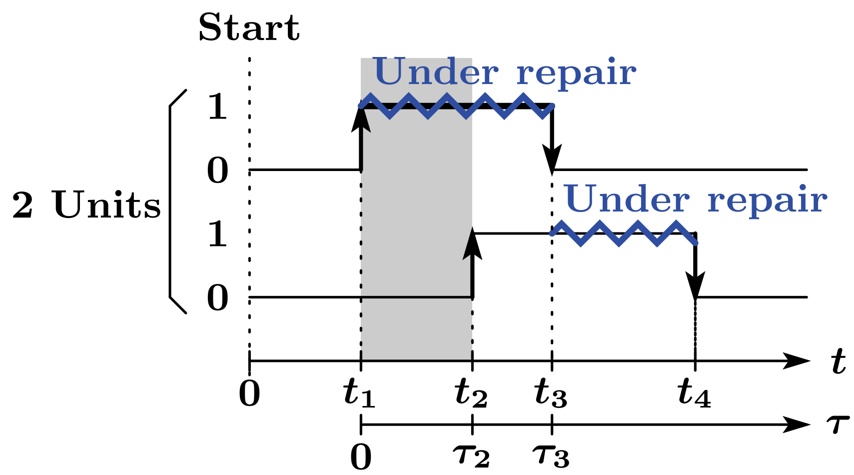
Unavailability $\approx [t_2, t_3]$ in Scenario $[0, t_4]$

4. Analysis based on scenario including awaiting repair (2/6)



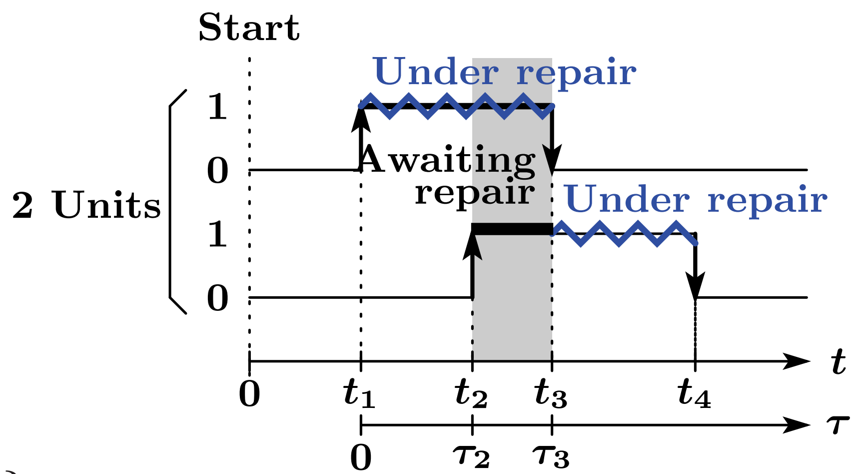
$$
\begin{aligned}
T_1 &= \mathrm{E}\{t_1\} \\
&= \int_0^\infty t \cdot \Pr\{\text{1st failure at } t\}\, dt \\
&= \int_0^\infty t \cdot \Pr\{\text{failure in one at } t\} \\
&\qquad \times \Pr\{\text{another unit is normal until } t\}\, dt \\
&= \int_0^\infty t \cdot 2\lambda e^{-\lambda t} \cdot e^{-\lambda t} dt \\
&= \frac{1}{2\lambda}
\end{aligned}
$$

4. Analysis based on scenario including awaiting repair (3/6)



$$
\begin{aligned}
T_2 &= \mathrm{E}\{\tau_2\} \\
&= \int_0^\infty \tau \cdot \Pr\{\text{2nd failure at } \tau\} \\
&\qquad \times \Pr\{\text{repair for 1st failure from } \tau = 0 \\
&\qquad\qquad \text{doesn't finish until } \tau\}\, d\tau \\
&= \int_0^\infty \tau \cdot \lambda e^{-\lambda\tau} \cdot e^{-\mu\tau} d\tau \\
&= \frac{\lambda}{(\lambda + \mu)^2}
\end{aligned}
$$

4. Analysis based on scenario including awaiting repair (4/6)



$$T_3 = \mathrm{E}\{\tau_3 - \tau_2\}$$

$$= \int_0^\infty d\tau_2 \int_{\tau_2}^\infty d\tau \cdot (\tau - \tau_2) \cdot \Pr\{\text{2nd failure at } \tau = \tau_2\}$$

$$\times \Pr\{\text{repair for 1st failure from } \tau = 0 \text{ finishes at } \tau\}$$

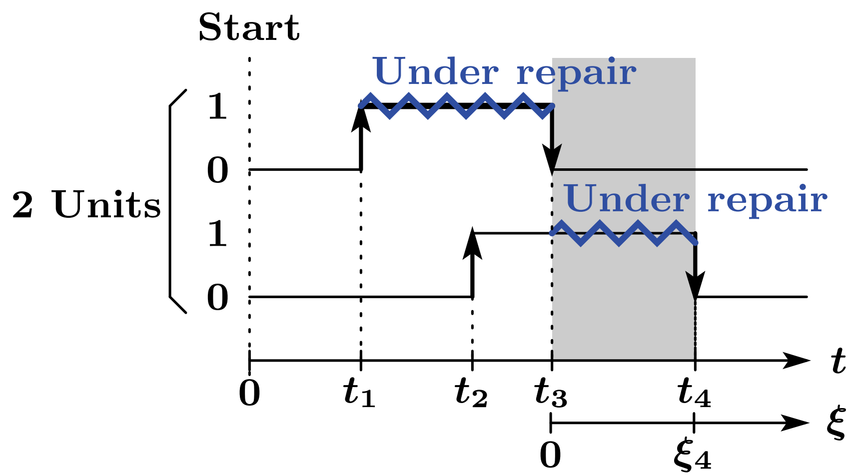
$$= \int_0^\infty d\tau_2 \int_{\tau_2}^\infty d\tau \cdot (\tau - \tau_2) \cdot \lambda e^{-\lambda \tau_2} \cdot \mu e^{-\mu \tau}$$

$$= \frac{\lambda}{\mu(\lambda + \mu)}$$

Repair time : $T_2 + T_3 = \frac{1}{\mu}\left[1 - \left(\frac{\mu}{\lambda + \mu}\right)^2\right] < \frac{1}{\mu}$ (solo operation case)

$\longleftarrow$ under the condition : 2nd failure during repair

4. Analysis based on scenario including awaiting repair (5/6)



$$
\begin{aligned}
T_4 &= \mathrm{E}\{\xi_4\} \\
&= \int_0^\infty \xi \\
&\qquad \times \Pr\{\text{repair for 1st failure from } \xi = 0 \text{ finishes at } \xi\} \\
&\qquad \times \Pr\{\text{unit restarting at } \xi = 0 \text{ is normal until } \xi\}\, d\xi \\
&= \int_0^\infty \xi \cdot \mu e^{-\mu\xi} \cdot e^{-\lambda\xi}\, d\xi \\
&= \frac{\mu}{(\lambda+\mu)^2}
\end{aligned}
$$

4. Analysis based on scenario including awaiting repair (6/6)

Mean unavailability in Scenario $[0, t_4]$

$$\frac{T_3}{T_1 + T_2 + T_3 + T_4} = \frac{2\lambda^2}{2\lambda^2 + 3\lambda\mu + \mu^2}$$

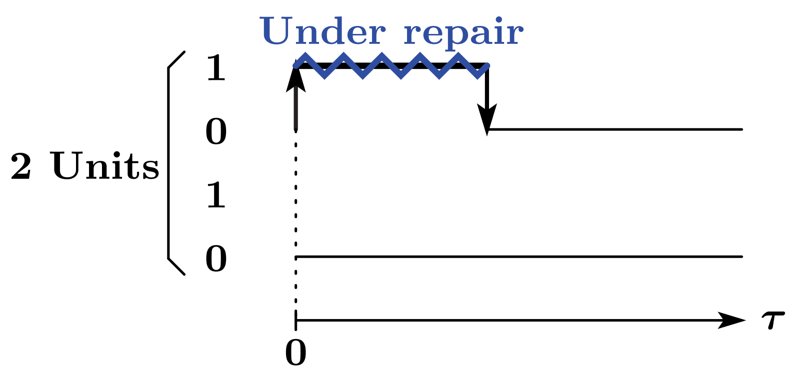
⬇ **Approximation condition**

Asymptotic mean unavailability $U = \frac{2\lambda^2}{2\lambda^2 + 3\lambda\mu + \mu^2}$

cf. $U_{\mathrm{Markov}} = \frac{2\lambda^2}{2\lambda^2 + 2\lambda\mu + \mu^2}$

4. Analysis based on scenario including awaiting repair — Mean repair time — (1/2)

(i) 2nd failure doesn't occur during repair



$$\mathrm{E}\{\text{repair time} \mid \text{2nd failure doesn't occur during repair}\}$$

$$= \int_0^\infty \tau \times \Pr\{\text{repair for 1st failure finishes at } \tau$$

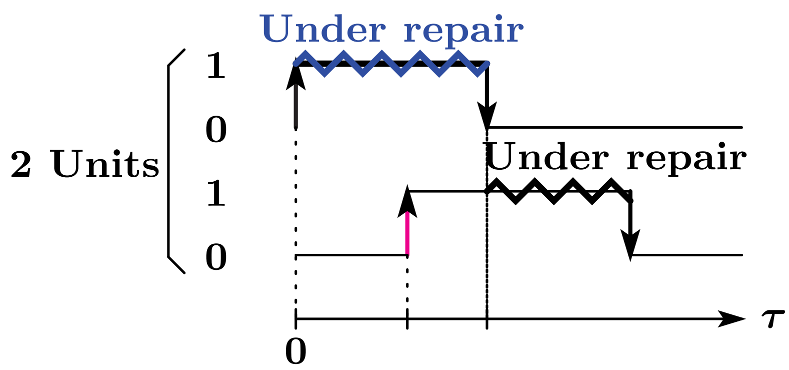
$$\times \Pr\{\text{another unit is normal until } \tau\}\, d\tau$$

$$= \int_0^\infty \tau \cdot \mu e^{-\mu\tau} \cdot e^{-\lambda\tau}\, d\tau$$

$$= \frac{\mu}{(\lambda+\mu)^2}$$

4. Analysis based on scenario including awaiting repair — Mean repair time — (2/2)

(ii) 2nd failure during repair



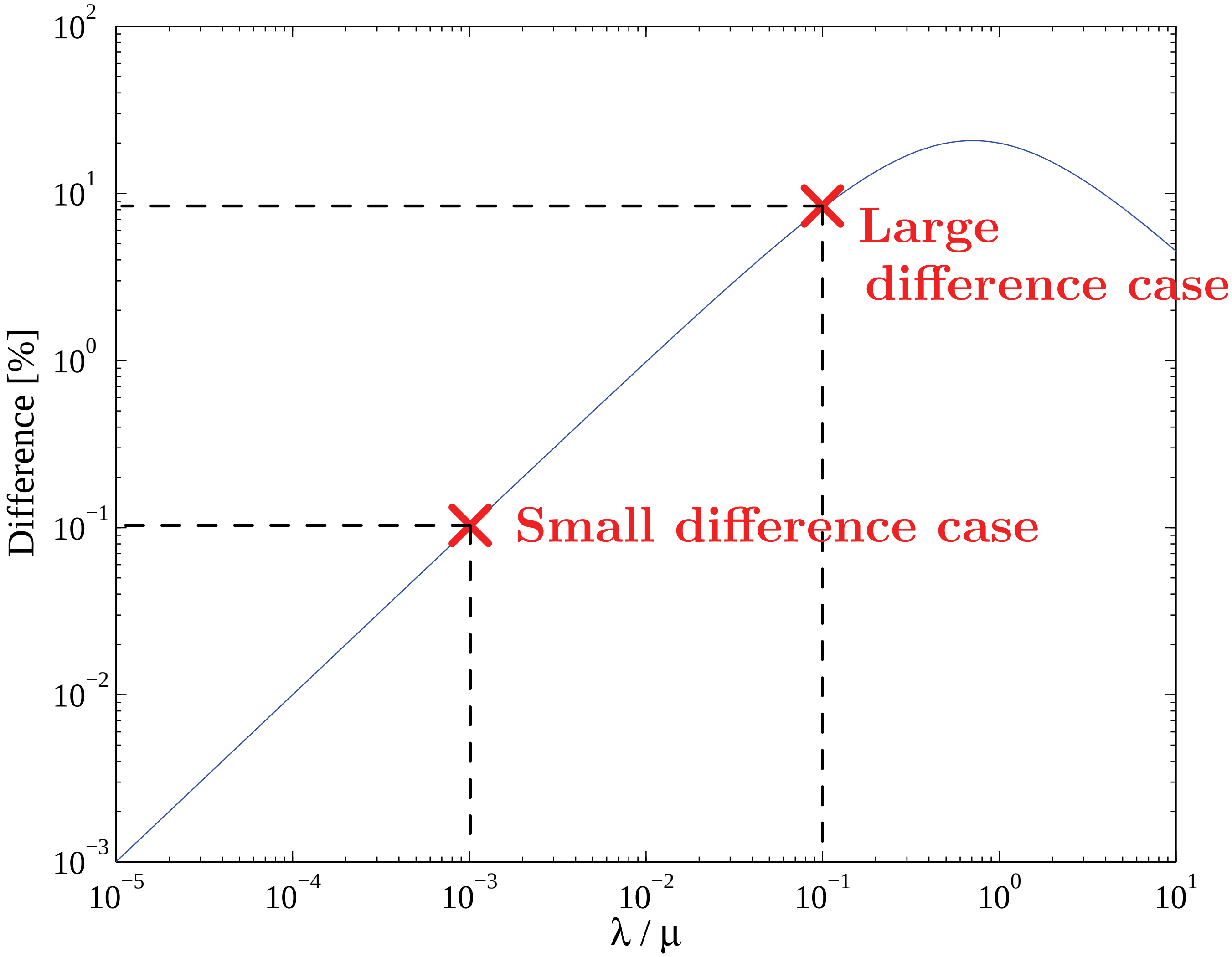
$$\mathrm{E}\{\text{repair time} \mid \text{2nd failure during repair}\} = T_2 + T_3$$

⬇

$$\begin{aligned}
&\text{Mean repair time} \\
&= \mathrm{E}\{\text{repair time} \mid \text{2nd failure doesn't occur during repair}\} \\
&\quad + \mathrm{E}\{\text{repair time} \mid \text{2nd failure during repair}\} \\
&= \frac{\mu}{(\lambda+\mu)^2} + (T_2 + T_3) = \frac{1}{\mu}
\end{aligned}$$

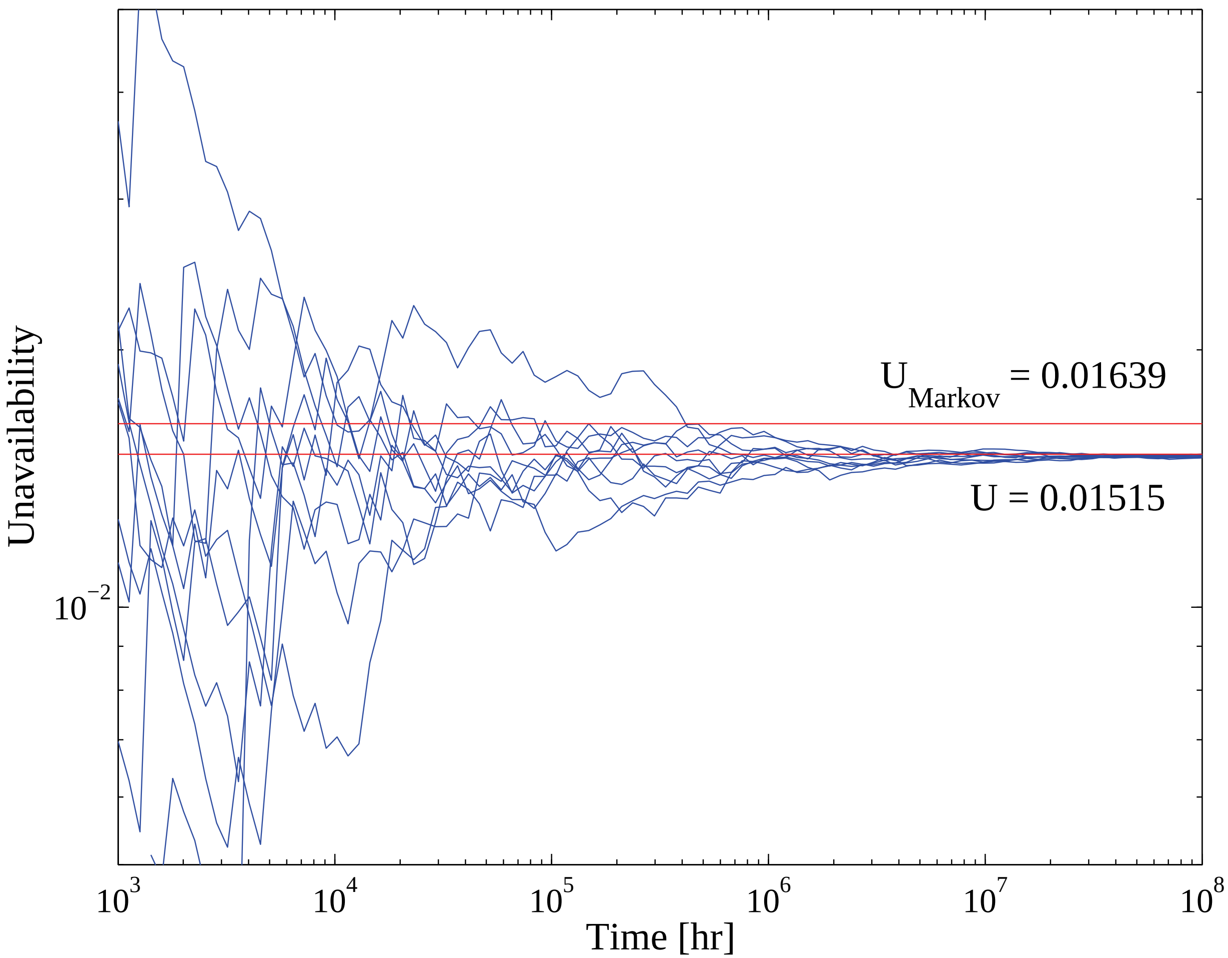
5. Monte Carlo simulations (1/3)

$$U_{\mathrm{Markov}} = \frac{2\lambda^2}{2\lambda^2 + 2\lambda\mu + \mu^2} > U = \frac{2\lambda^2}{2\lambda^2 + 3\lambda\mu + \mu^2}$$



5. Monte Carlo simulations (2/3)

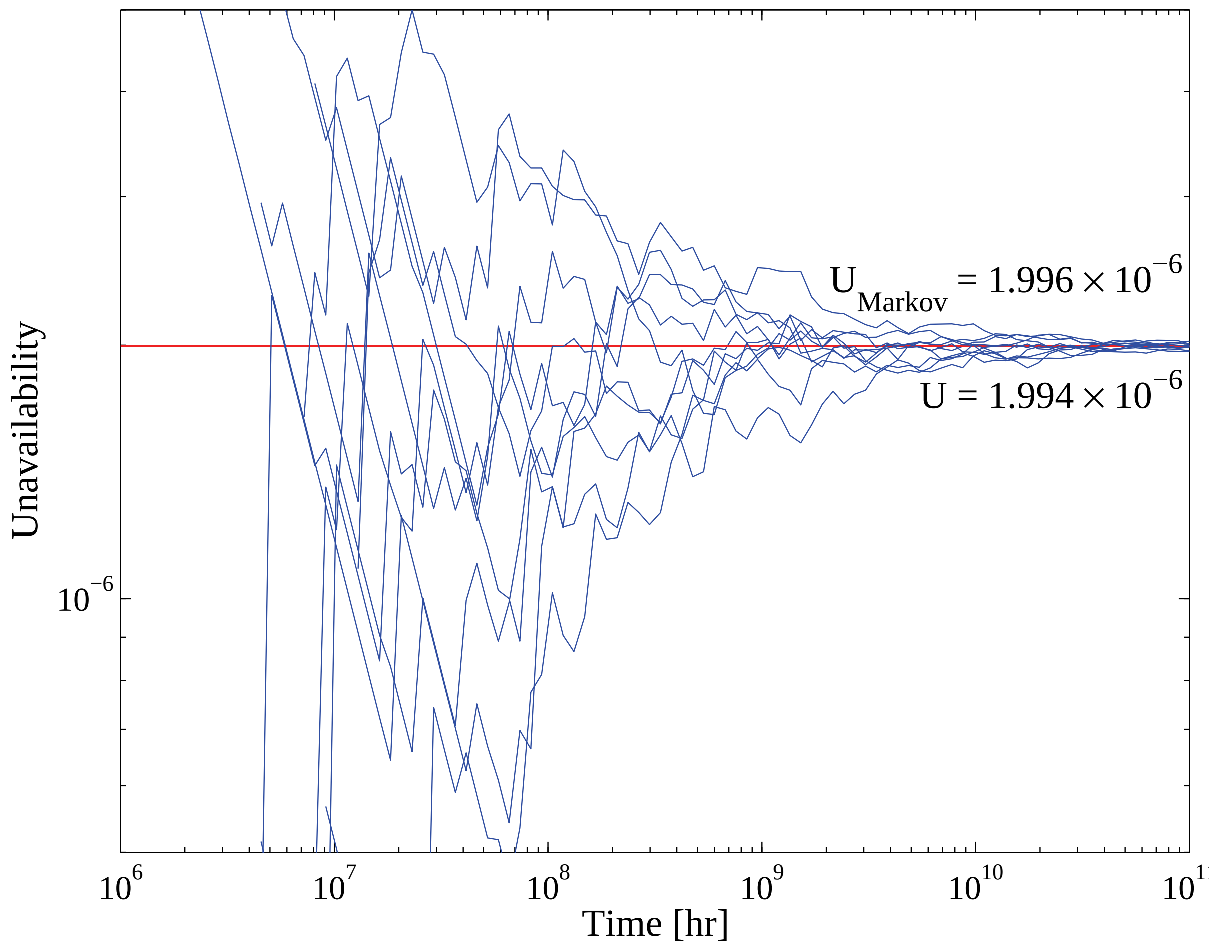
(i) Large difference (8%) case
($\lambda = 1 \times 10^{-2}$[/hr], $\mu = 1 \times 10^{-1}$[/hr])



U is more reasonable than U_{Markov}

5. Monte Carlo simulations (3/3)

(ii) Small difference (0.1%) case
($\lambda = 1 \times 10^{-4}$[/hr], $\mu = 1 \times 10^{-1}$[/hr])



$$\approx 2\left(\frac{\lambda}{\mu}\right)^2$$

U_{Markov} is of practical use only when $\lambda \ll \mu$

6. Conclusion

- **New unavailability formula for 1-out-of-2 system with one repair team**
- **Its validity confirmed by Monte Carlo simulations**

⬇

One solution to problem of Markov analysis

Future works

- **more complicated redundant systems**
- **another distribution than exponential**
  (ex. log-normal distribution)