DATA ANALYSIS FOR FIRE FREQUENCY

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This short paper addresses the application Bayes' Theorem in assessing frequency of rare events in a QRA.

A key element of risk is randomness (i.e., uncertainty) - not knowing exactly when, where, or if damage, or a tragic occurrence is going to happen. Risk analysis uses various methods of modelling, analysis, and evaluation, and thus contains various types of uncertainties. In general, these uncertainties may be attributable to a number of factors, such as:

1. The statistical nature of data,
2. Insufficient understanding of physical and biological phenomena
3. Unpredictable events (e.g., natural, biological and human behaviour)

The information derived from a risk analysis may or may not closely reflect reality due to the fact that they are predicated on idealized assumptions or conditions. This information must often be inferred from similar (or even different) circumstances or derived through modelling, and thus may be in various degrees of imperfection (i.e., uncertainty). Many problems in risk analysis involve natural processes and phenomena that are inherently random, and the states of such phenomena are naturally indeterminate and thus cannot be described with definiteness. Therefore, idealized assumptions or conditions containing a certain amount of uncertainties are inferred.

Most rare failure events, such as fires in a complex engineering system, may seldom occur in the system's operating history, we must then use generic databases from other similar systems to supplement the lack of system-specific experience. We must note that it is incorrect to use generic database without justification or adjustment. For example, it is not reasonable to apply the failure rate of certain system from UK for a HK system, which, in fact, uses a different design and maintenance practice.

Following the Bayesian data update methodology [Ref. 1-3], we would combine both system-specific experience, and other system's experience or generic data. We should note that no previous event occurred in a system does not mean no data. If we have no failure occur in the past 20 years in a system, we have a data point saying 0 event in 20 years. It is very different from no data.

Following Reference 1, the Bayes' Theorem can be used to assessing fire frequency, λ,

$$\pi'(\lambda \mid E) = \frac{\pi(\lambda)\, L(E \mid \lambda)}{\int_0^\infty d\lambda\, \pi(\lambda)\, L(E \mid \lambda)} \qquad (1)$$

Where

$\pi'(\lambda\|E)$	=	posterior distribution, the probability density function of λ given evidence of E
$\pi(\lambda)$	=	prior distribution, the probability density function of λ prior to having evidence of E
$L(E\|\lambda)$	=	likelihood function, the probability of the evidence given λ

If the likelihood function is modelled by the Poisson distribution and the prior is a gamma distribution, then the posterior distribution would also be a gamma distribution. A gamma distribution is

$$\pi(\lambda) = \frac{\beta^\alpha}{\Gamma(\alpha)}\, \lambda^{\alpha-1}\, e^{-\beta\lambda} \qquad (2)$$

where α and β are the two parameters of the distribution. The posterior distribution will take the same form as Equation 2 with parameters:

$$\alpha' = \alpha + r; \quad \beta' = \beta + T \tag{2.14}$$

where

r = number of fires in the evidence

T = number of years covered by the evidence

The evidence (r, T) would be the actual system experience. In typical frequentist calculation, fire frequency $\lambda = r/T$. But this process would break down if r=0.

Using equations 1 and 2, we can process data with both system specific experience and generic data such that posterior $\lambda = \alpha'/\beta'$. Thus, an operating history of zero event (r=0) would also lead to a meaningful evidence and a posterior λ because it is not likely that α= 0 in the generic database. The process can be repeated as necessary if further evidence is available, and there are also other techniques to be used to specialise data from expert opinion, pooled data, biased data, incomplete data, etc.

Furthermore, several equations might be used to calculate the unavailability of an event in a QRA, depending on the circumstances. For instance, the following equation would be used to calculate the failure probability of an operating component without repair in a non-demand failure mode:

$$P = 1 - e^{-\lambda t_m} \tag{3}$$

Where

P = failure probability of basic event

λ = failure rate per hour

t_m = mission time expressed in hours

For a standby component with non-demand failure mode and periodic testing, the failure probability is

$$P = 1 + \frac{e^{-\lambda T} - 1}{\lambda T} \tag{4}$$

Where

P = failure probability of basic event

T = average time to repair expressed in hours

For an operating component given to the ability to repair the component, the failure probability is:

$$P = \frac{\lambda T}{1 + \lambda T}\left(1 - e^{-(\lambda + \frac{1}{T})t_m}\right) \tag{5}$$

All these equations require the knowledge of the failure frequency, λ, and Bayes' Theorem (sometimes, two-stages or multi-stages Bayesian update) would be often used to assess the posterior λ by combining both system specific experience and generic data.

Reference

1. G. Apostolakis and M. Kazarians, "The Frequency of Fires in Light Water Reactor Compartments, Presented at the ANS/ENS Topical Meeting on Thermal Reactor Safety, Knoxville, Tennessee, April 7-11, 1980.
2. J.K. Vaurio, "On Analytical Empirical Bayes Estimation of failure Rates," Risk Analysis, Vol. 7, No. 3, 1987.
3. G. Apostolakis, "Data Analysis in Risk Assessments," Nuclear Engineering and Design, 71(1982) 375-381.