CONSIDERATION OF UNCERTAINTY IN STRUCTURAL DESIGN OF RC FRAME SUBJECTED TO SEISMIC LOADING

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- Response of structure to seismic loading
- Fuzzy numbers and fuzzy arithmetic
- Response surface function
- Numerical example

Response of Structure to Seismic Loading

General governing equation:

$$\boldsymbol{M}\frac{d^2\boldsymbol{d}(t)}{dt^2}+\boldsymbol{C}\frac{d\boldsymbol{d}(t)}{dt}+\boldsymbol{K}\boldsymbol{d}(t)=\boldsymbol{f}(t)$$

$\boldsymbol{M}$	mass matrix
$\boldsymbol{C}$	damping matrix
$\boldsymbol{K}$	stiffness matrix
$\boldsymbol{f}(t)$	load vector
$\boldsymbol{d}(t)$	vector of nodal displacements

Response of Structure to Seismic Loading

Single Degree Subjected to Seismic Loading - Response Spectrum:

$$\ddot{v}(t) + 2\,\xi\,\omega\,\dot{v}(t) + \omega^2\,v(t) = f\,\ddot{v}_g(t)$$

$v(t)$	relative displacement
$\ddot{v}_g(t)$	ground acceleration
ω	natural frequency
ξ	damping
f	mode participation factor

Response of Structure to Seismic Loading

Duhamel Integral - Response Spectrum:

$$v(t) = \frac{f}{\omega}\int_0^t -\ddot{v}_g(\tau)\sin\omega(t-\tau)e^{-\xi\omega(t-\tau)}d\tau$$

τ time of load application

Displacement Response Spectrum:

$$S_d(\omega) = v(\omega)_{\max}$$

Velocity Response Spectrum:

$$S_v(\omega) = \omega S_d(\omega)$$

Acceleration Response Spectrum:

$$S_a(\omega) = \omega^2 S_d(\omega)$$

Response of Structure to Seismic Loading

Natural Vibration:

$$\left(\boldsymbol{K} - \omega^2 \boldsymbol{M}\right)\boldsymbol{u} = \boldsymbol{0}$$

$\boldsymbol{u}$ eigenmode

Vibration of Structure Induced by Seismic Loading:

$$\boldsymbol{M}\,\ddot{\boldsymbol{d}}(t) + \boldsymbol{C}\,\dot{\boldsymbol{d}}(t) + \boldsymbol{K}\,\boldsymbol{d}(t) = -\boldsymbol{M}\,\boldsymbol{s}\,\ddot{\boldsymbol{v}}_g(t)$$

$\boldsymbol{s}$ horizontal or vertical displacement

Unknown Displacements:

$$\boldsymbol{d}(t) = \boldsymbol{U}\,\boldsymbol{v}(t)$$

$\boldsymbol{U}$ matrix containing eigenmodes in its columns

Vibration of Structure Caused by Seismic Loading:

$$\boldsymbol{U}^T\,\boldsymbol{M}\,\boldsymbol{U}\,\ddot{\boldsymbol{v}}(t) + \boldsymbol{U}^T\,\boldsymbol{C}\,\boldsymbol{U}\,\dot{\boldsymbol{v}}(t) + \boldsymbol{U}^T\,\boldsymbol{K}\,\boldsymbol{U}\,\boldsymbol{v}(t) = -\boldsymbol{U}^T\boldsymbol{M}\,\boldsymbol{s}\,\ddot{\boldsymbol{v}}_g(t)$$

Response of Structure to Seismic Loading

Natural Vibration:

$$\ddot{v}_i(t) + 2\,\xi_i\,\omega_i\,\dot{v}_i(t) + \omega_i^{\,2}\,v_i(t) = f_i\,\ddot{v}_g(t)$$

i index denoting eigenmode

Maximum modal response of *i*-th period:

$$y(T_i)_{\max} = \frac{S_a(\omega_i)}{\omega_i^2}$$

T_i period of *i*-th mode

Maximum modal displacement of *i*-th mode:

$$\boldsymbol{d}_i = \left(\boldsymbol{u}_i^{\,T}\,\boldsymbol{M}\,\boldsymbol{s}\,y(T_i)_{\max}\right)\boldsymbol{u}_i$$

Response of Structure to Seismic Loading

Displacements for calculation of internal forces:

$$\boldsymbol{d} = \sum_{i=1}^{n} \left(y(T_i)_{\max} \boldsymbol{u}_i^T \boldsymbol{M} \boldsymbol{s} \right) \boldsymbol{u}_i$$

n number of eigenmodes

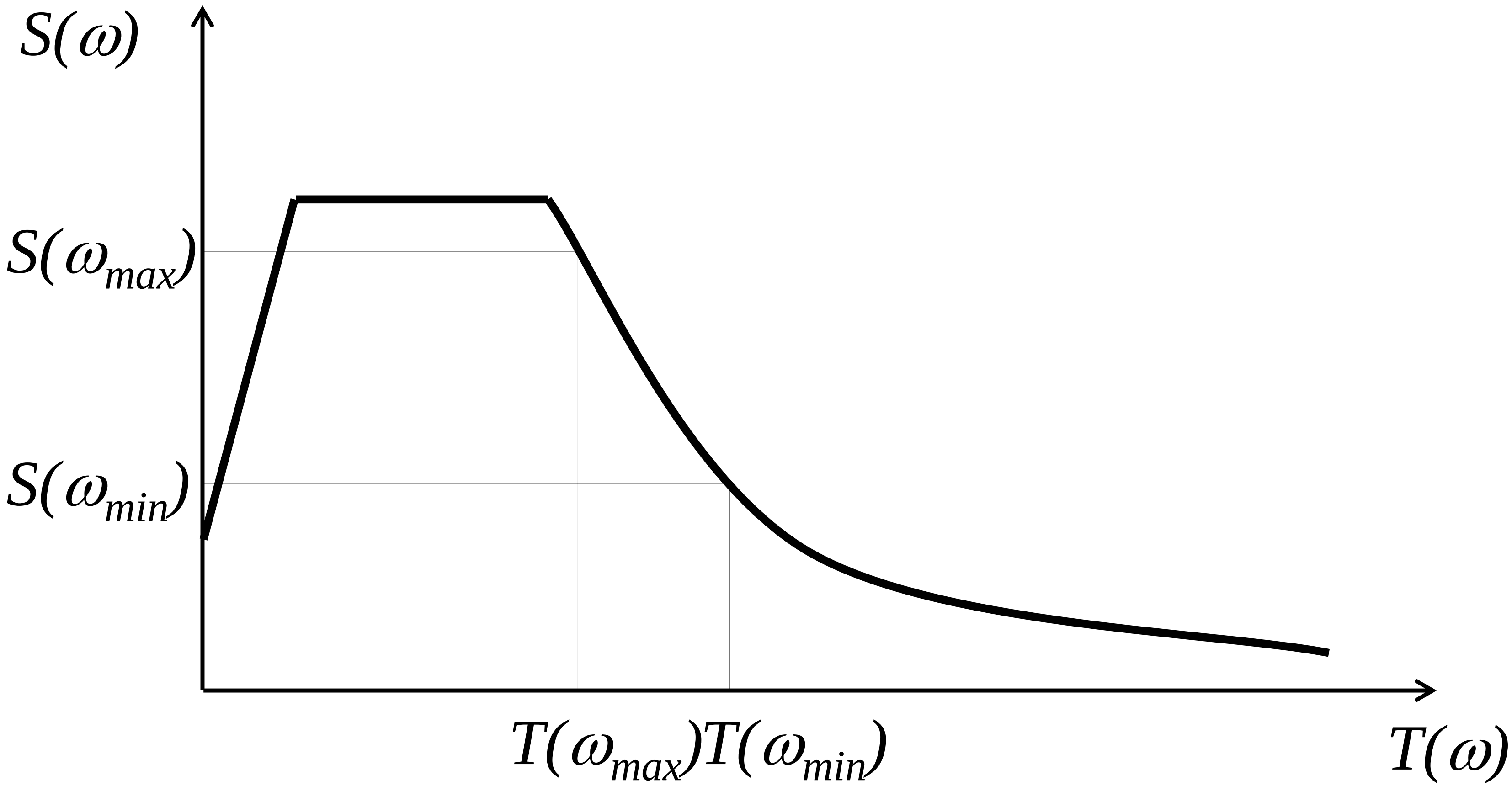
Equation for calculation of internal forces:

$$\boldsymbol{f} = \boldsymbol{K} \boldsymbol{d}$$

$\boldsymbol{K}$ stiffness matrix

Response of Structure to Seismic Loading

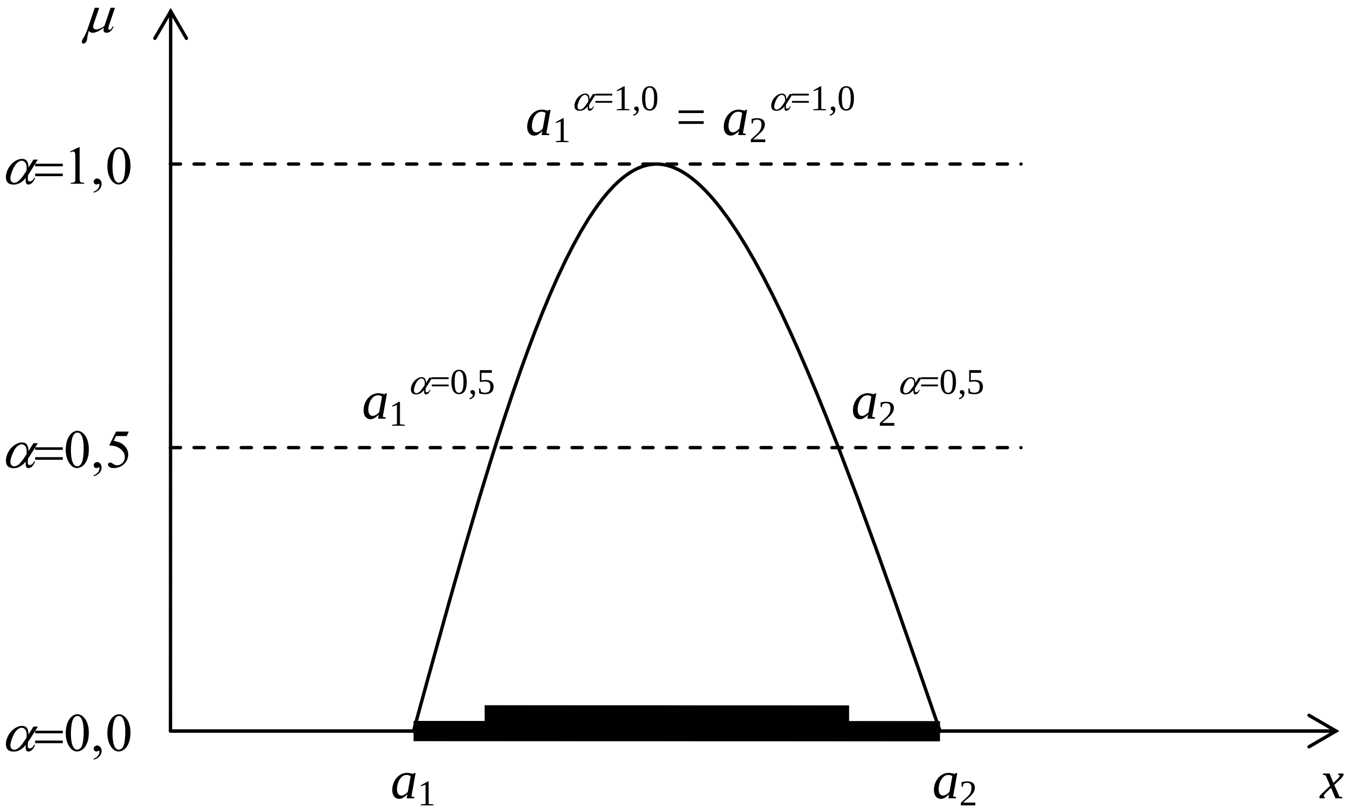
Example of response spectrum:



Fuzzy Numbers

A fuzzy number is a fuzzy set defined on the set of real numbers.

$$A = \{(x, \mu_A(x)) \mid x \in X\}$$

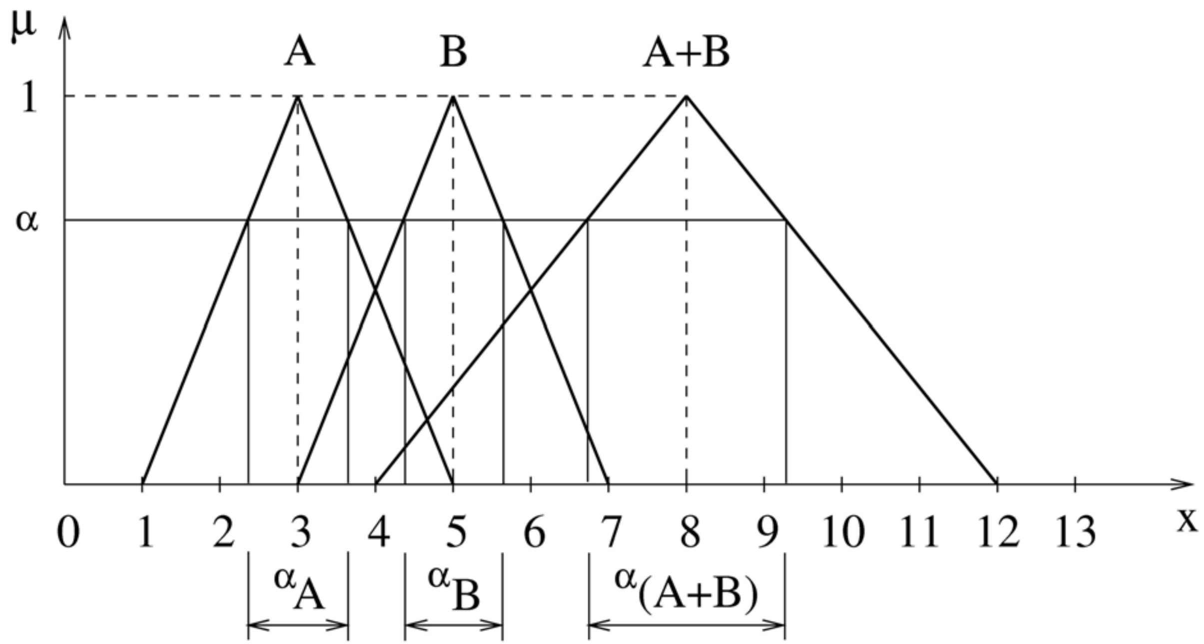


Fuzzy Arithmetic

Fuzzy operation are based on the extension principle

$$\mu_{A*B}(z) = \bigcup_{z=x*y} \left(\mu_A(x) \wedge \mu_B(y)\right)$$

$$^{\alpha}(A*B) = {}^{\alpha}A * {}^{\alpha}B \quad \longrightarrow \quad A*B = \bigcup_{\alpha\in[0,1]} {}^{\alpha}(A*B)$$



Fuzzy Arithmetic

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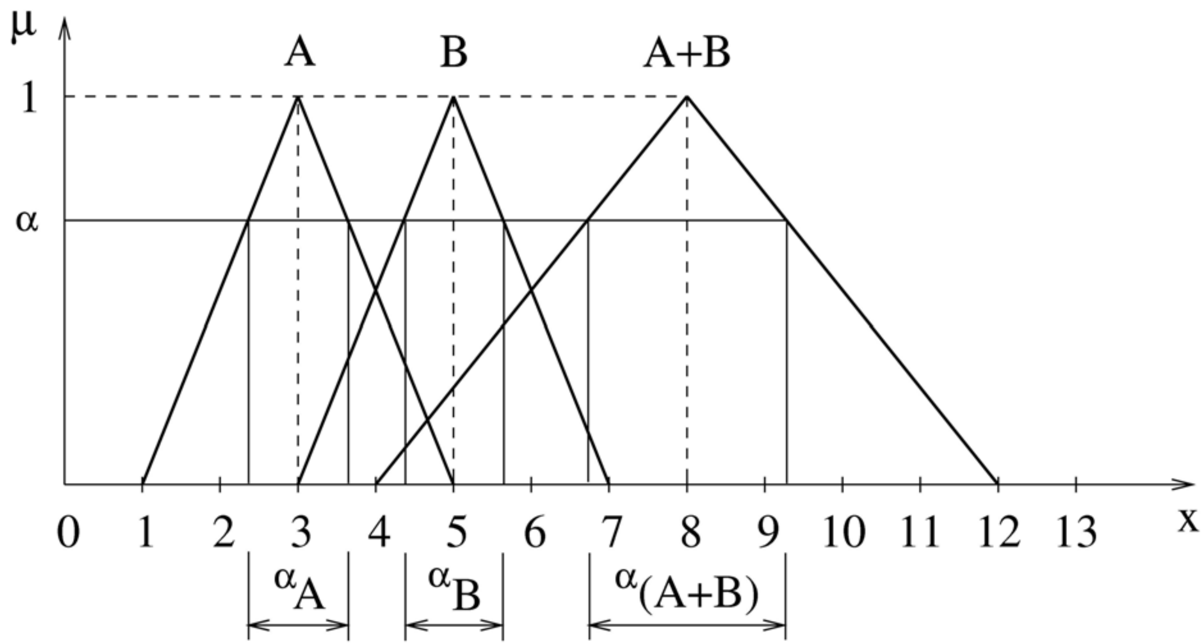


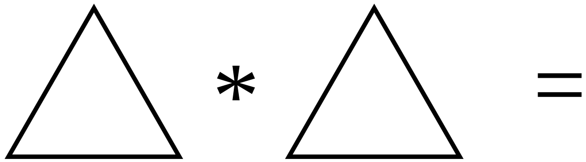
Arithmetic operations on α-cuts are interval arithmetic operations

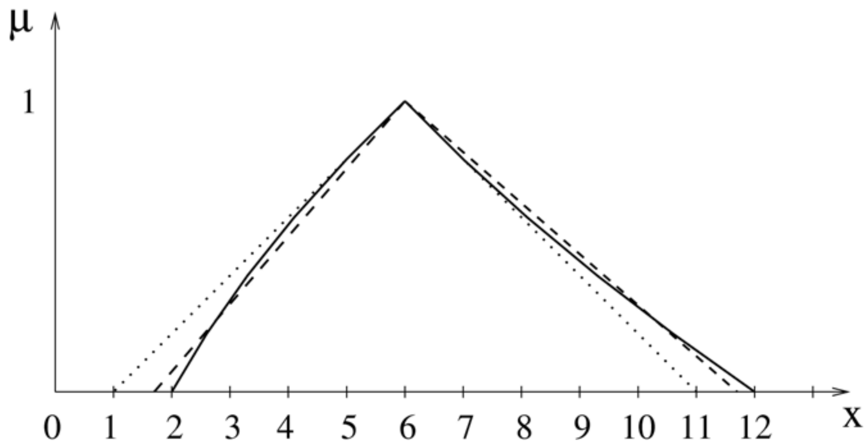
$$^{\alpha}A(+)^{\alpha}B = [\underline{a}_{\alpha}; \overline{a}_{\alpha}] + [\underline{b}_{\alpha}; \overline{b}_{\alpha}] = [\underline{a}_{\alpha} + \underline{b}_{\alpha}; \overline{a}_{\alpha} + \overline{b}_{\alpha}]$$

Fuzzy Arithmetic

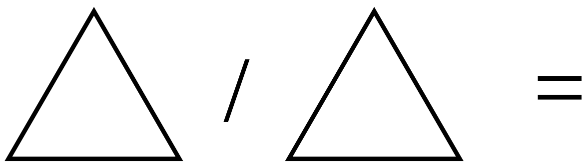
Examples of results of fuzzy arithmetic operations

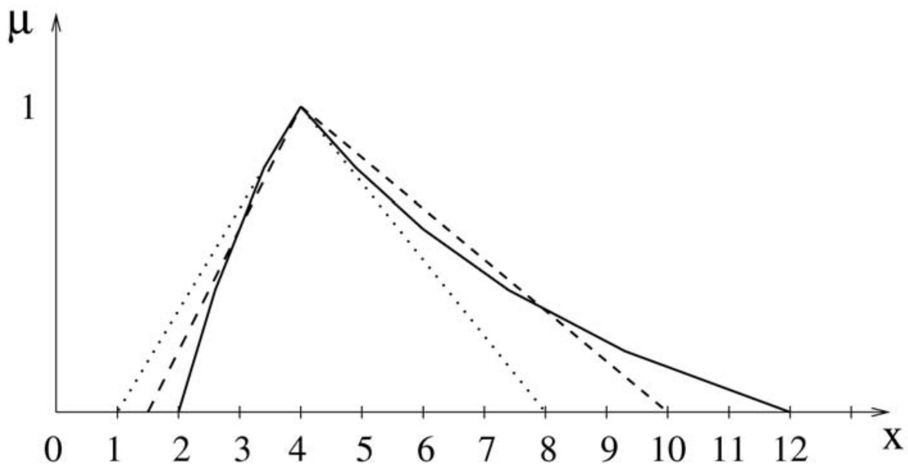
Multiplication:





Division:





Response Surface Function

General response of a structure:

$$\boxed{\tilde{\mathbf{y}} = \mathcal{F}(\tilde{\mathbf{x}}),}$$

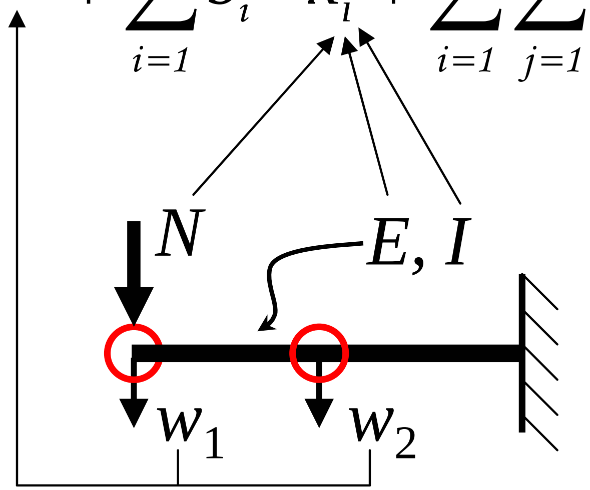
Input: $\tilde{\mathbf{x}} \in \tilde{\mathbf{X}}$

Output: $\tilde{\mathbf{y}} \in \tilde{\mathbf{Y}}$

Approximation of $\mathcal{F}$ in order to minimize necessary number of computation runs

$$f^{(k)}(x) = a^{(k)} + \sum_{i=1}^{n} b_i^{(k)} x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}^{(k)} x_i x_j,$$

Example:



Response Surface Function

Approximation of $\mathcal{F}$:

$$f^{(k)}(x) = a^{(k)} + \sum_{i=1}^{n} b_i^{(k)} x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^{(k)} x_i x_j,$$

Coefficients are obtained by the least square method.

$$F^{(k)}(a^{(k)}, b_i^{(k)}, c_{ij}^{(k)}) = \sum_{i=1}^{s} \left(f^{(k)}(\boldsymbol{x}^{[i]}) - y_k^{[i]} \right)^2.$$

In our case, the quadratic terms were omitted, which simplied further computation.

Numerical Example

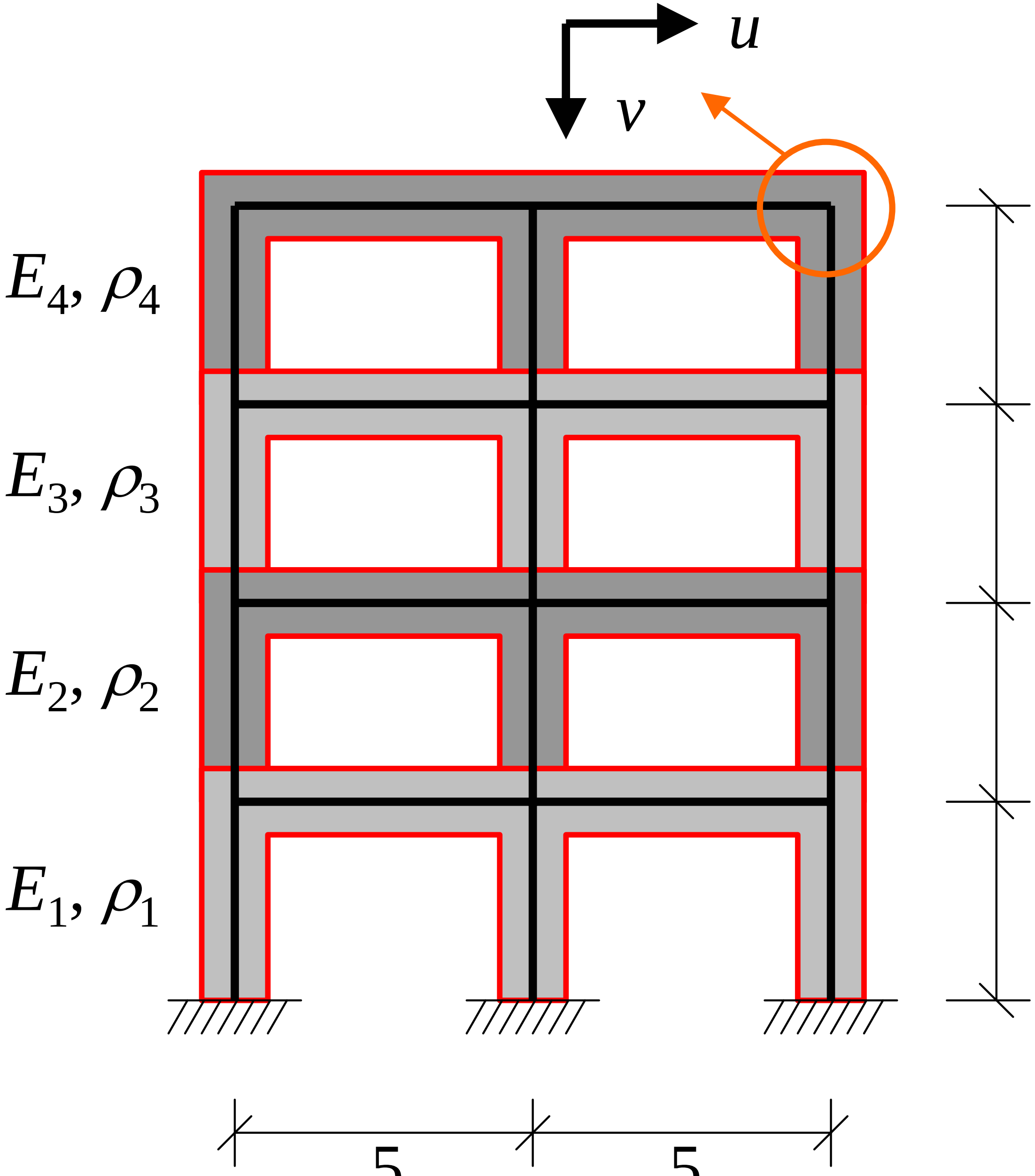
RC 2D frame:

Modulus of elasticity:
E = 30 GPa

Density:
$\rho = 2500$ kg/m^3

Quantities vary by ± 10%

△ fuzzy numbers



Numerical Example

Objective of analysis:

First 5 natural vibration modes of 2D frame
(5 frequencies and 5 mode shapes).

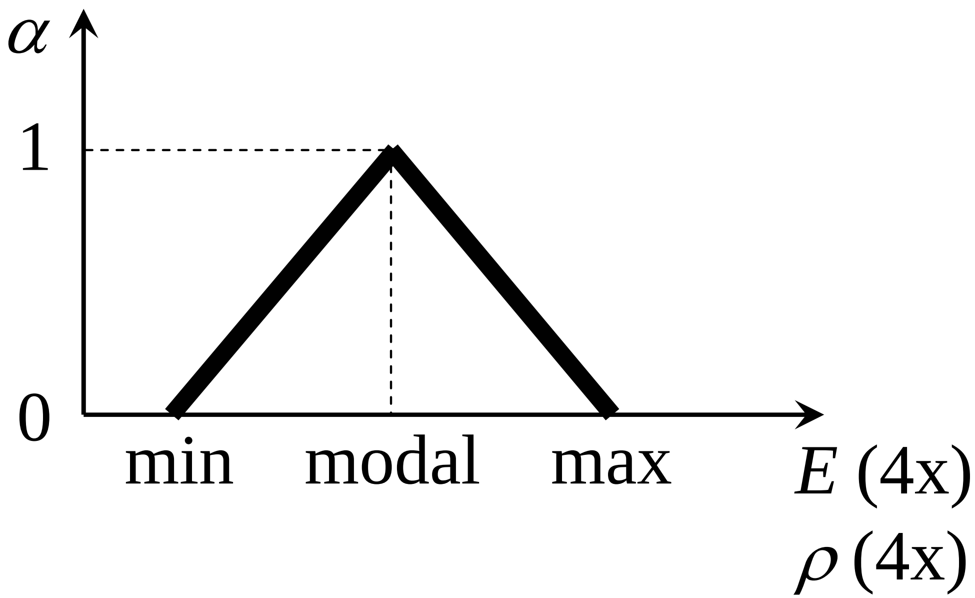
12	joints
x 2	displacements at joint
x 5	natural modes
120	
+ 5	natural frequencies
125	response surface functions

$$f^{(k)}(x) = b_1^{(k)}E_1 + b_2^{(k)}E_2 + b_3^{(k)}E_3 + b_4^{(k)}E_4 + \\ + b_5^{(k)}\rho_1 + + b_6^{(k)}\rho_2 + b_7^{(k)}\rho_3 + b_8^{(k)}\rho_4 + b_9^{(k)}.$$

Numerical Example

Coefficients of surface response function:

$$f^{(k)}(x) = b_1^{(k)} E_1 + b_2^{(k)} E_2 + b_3^{(k)} E_3 + b_4^{(k)} E_4 + \\ + b_5^{(k)} \rho_1 + + b_6^{(k)} \rho_2 + b_7^{(k)} \rho_3 + b_8^{(k)} \rho_4 + b_9^{(k)}.$$

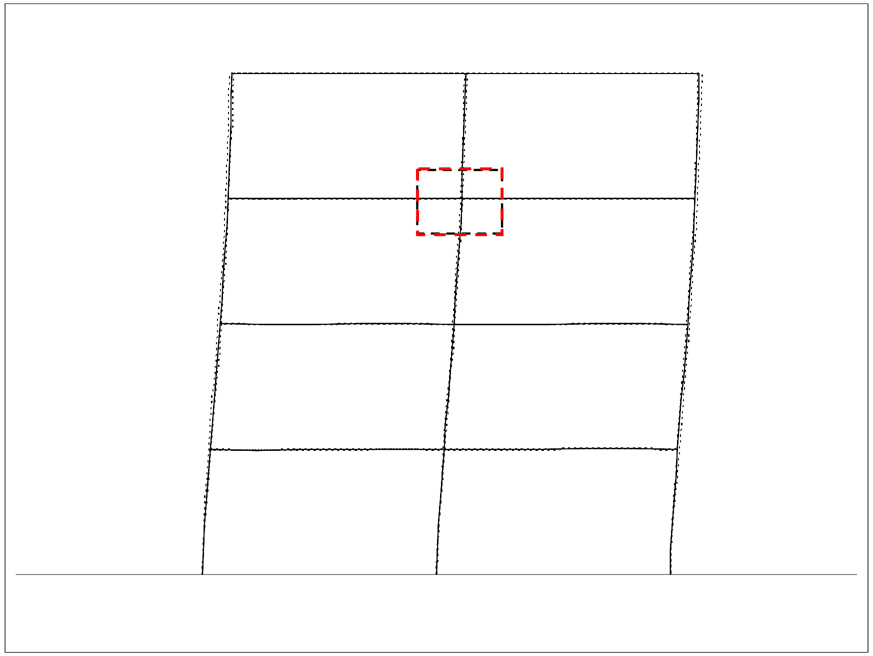


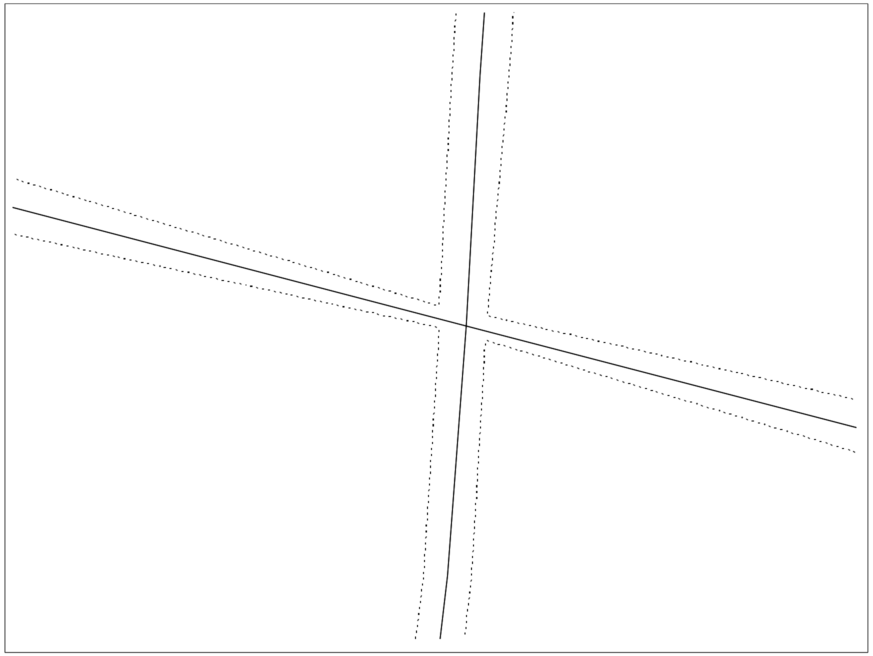
3^{2x4} = **6561** combinations (deterministic computation runs)

Numerical Example

Results:

Modal shape 1

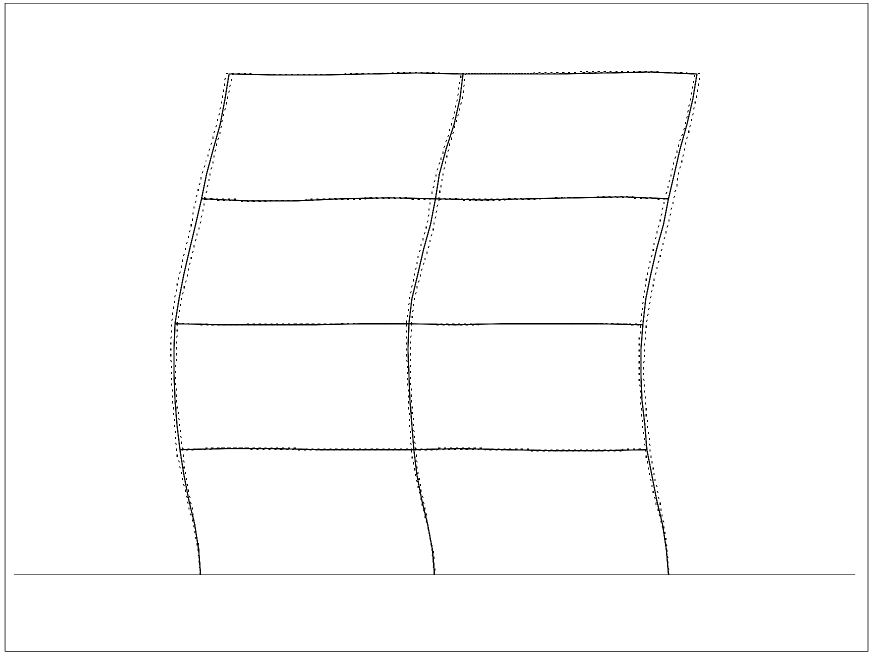


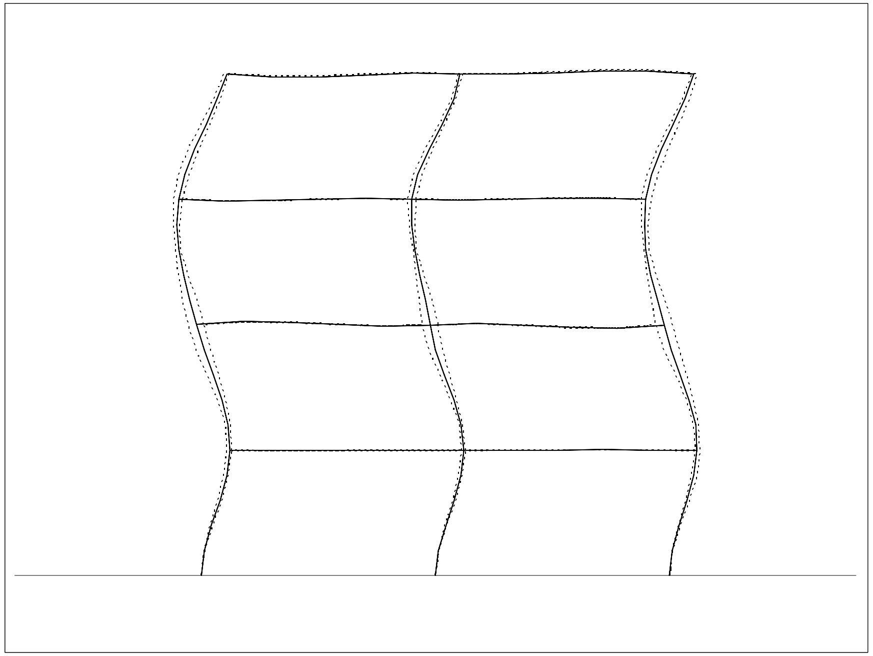


Numerical Example

Results:

Modal shape 2 and modal shape 3

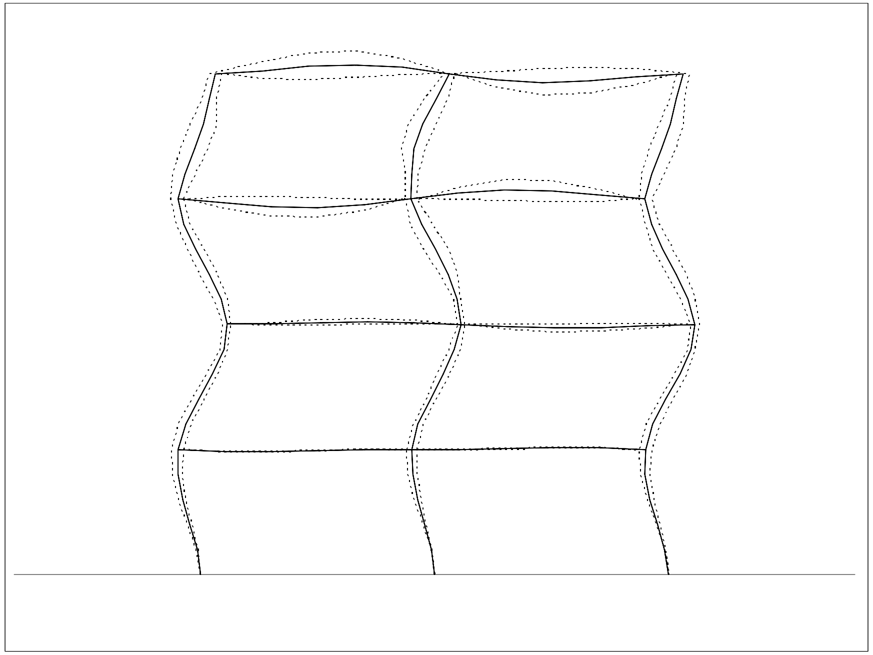


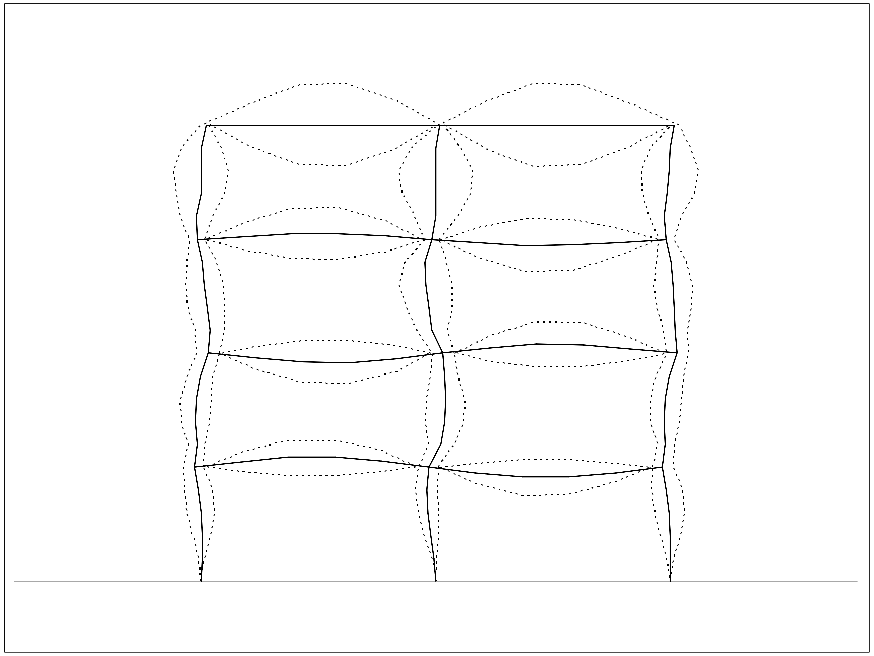


Numerical Example

Results:

Modal shape 4 and modal shape 5

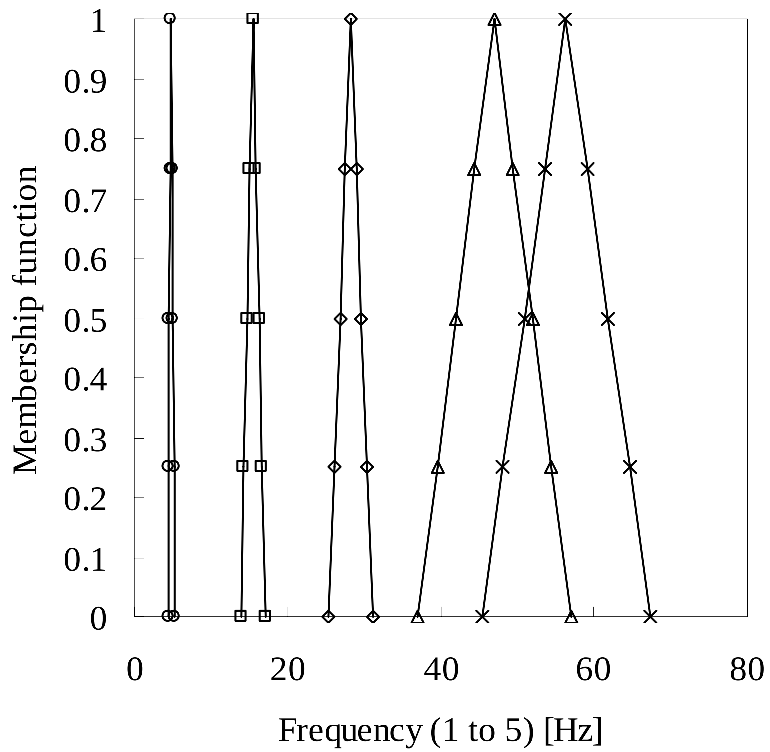


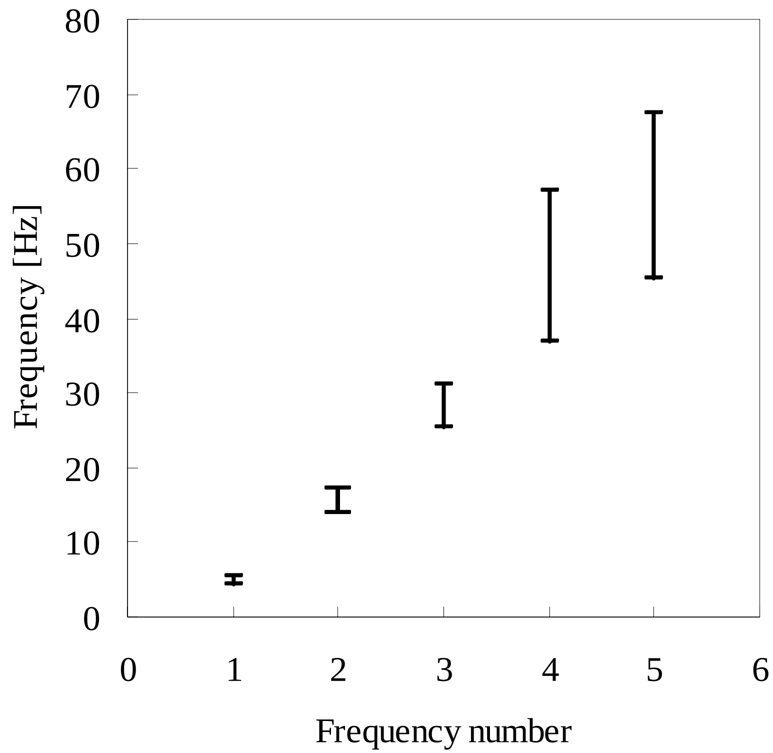


Numerical Example

Results:

First 5 natural frequencies

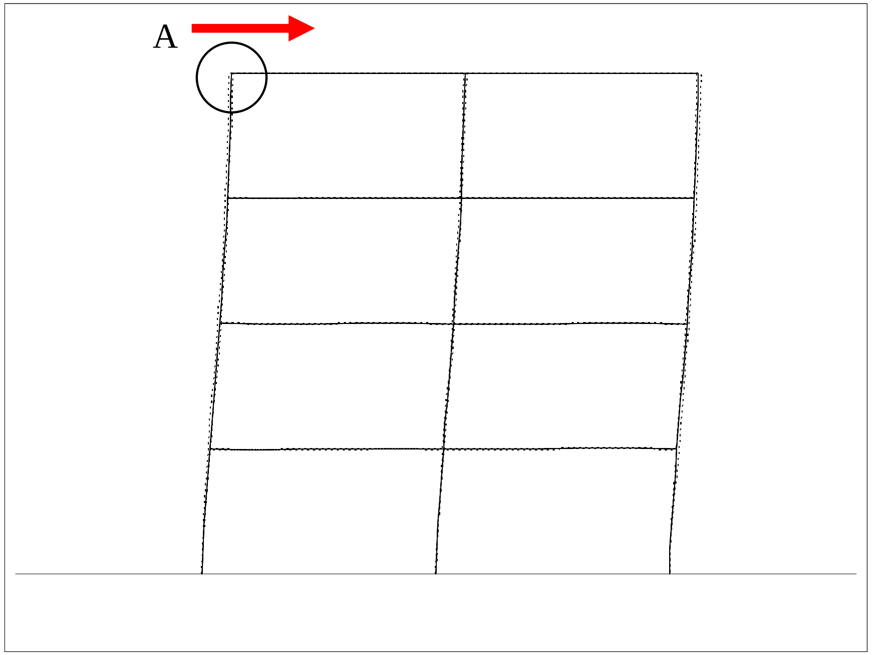




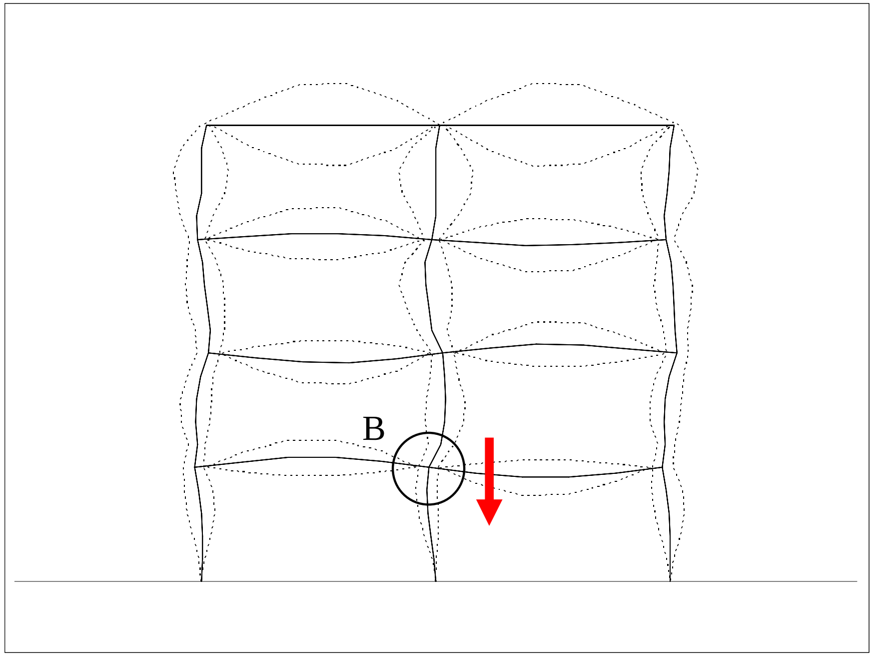
Numerical Example

Comparison btw response surface function and true fuzzy result:

Modal shape 1

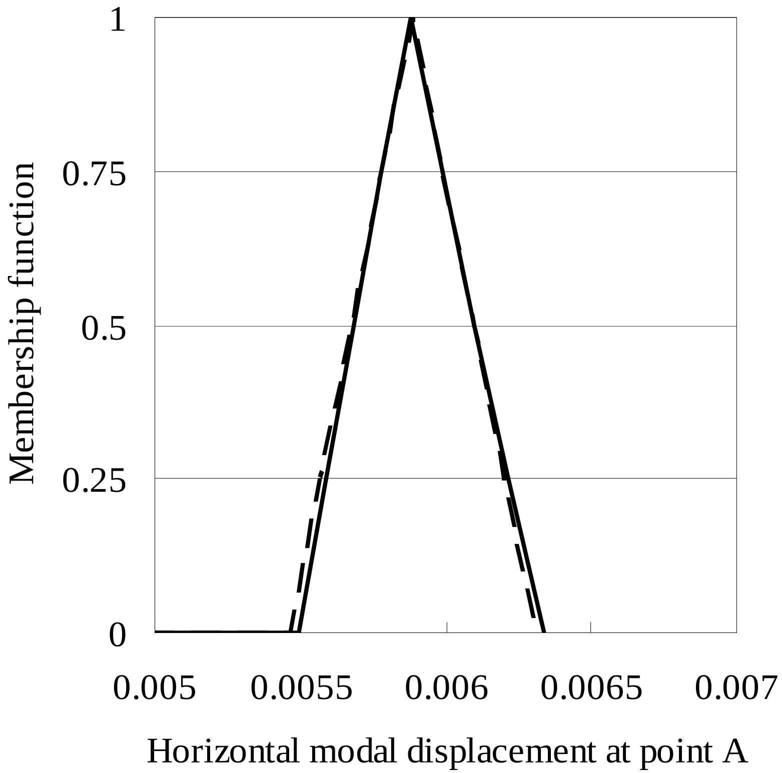


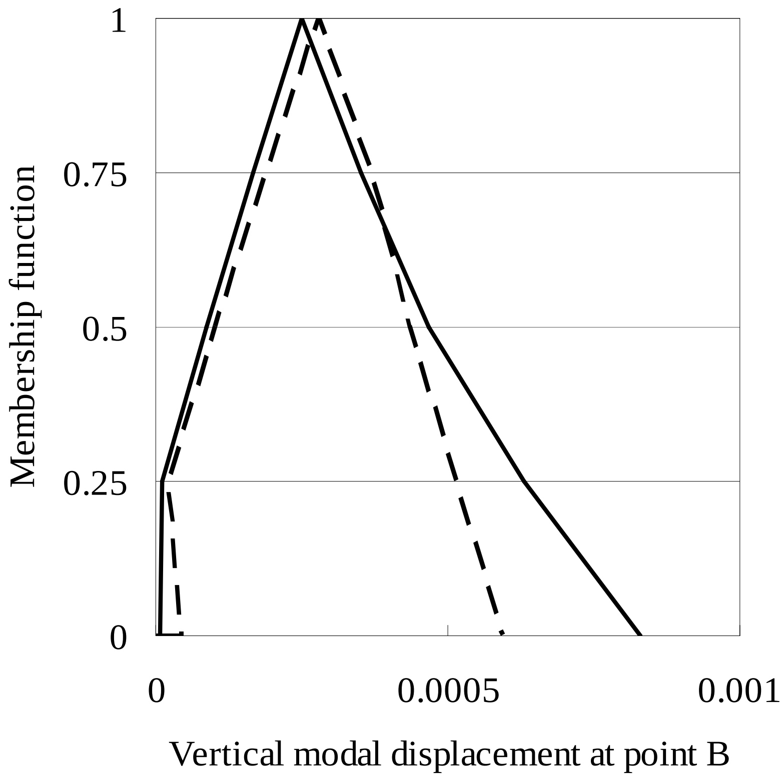
Modal shape 5



Numerical Example

Comparison btw response surface function and true fuzzy result:

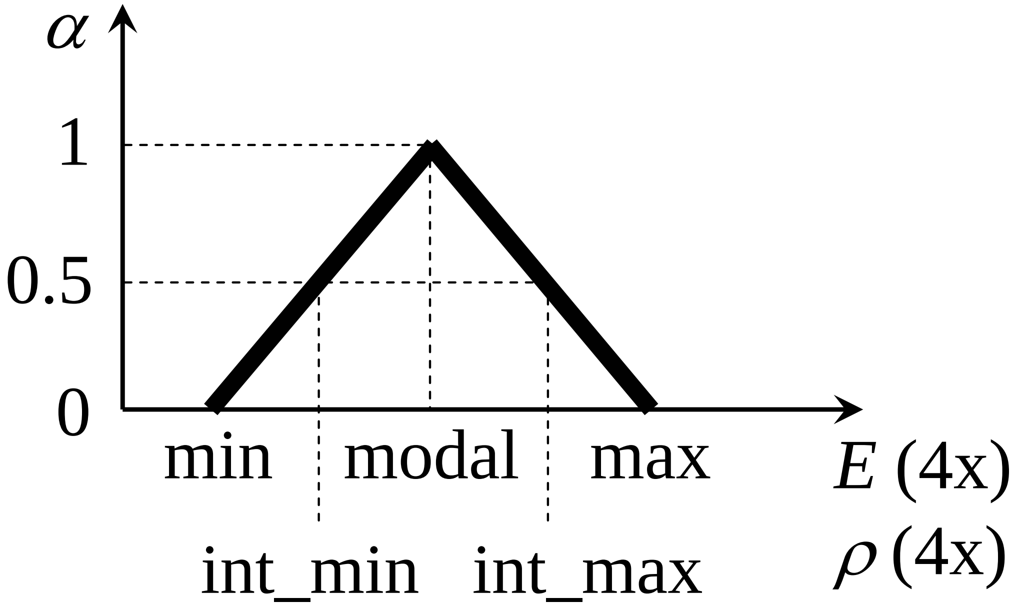




Numerical Example

Verification of necessary number of α-cuts:

3^{2x4} = **6561** combinations (deterministic computation runs)

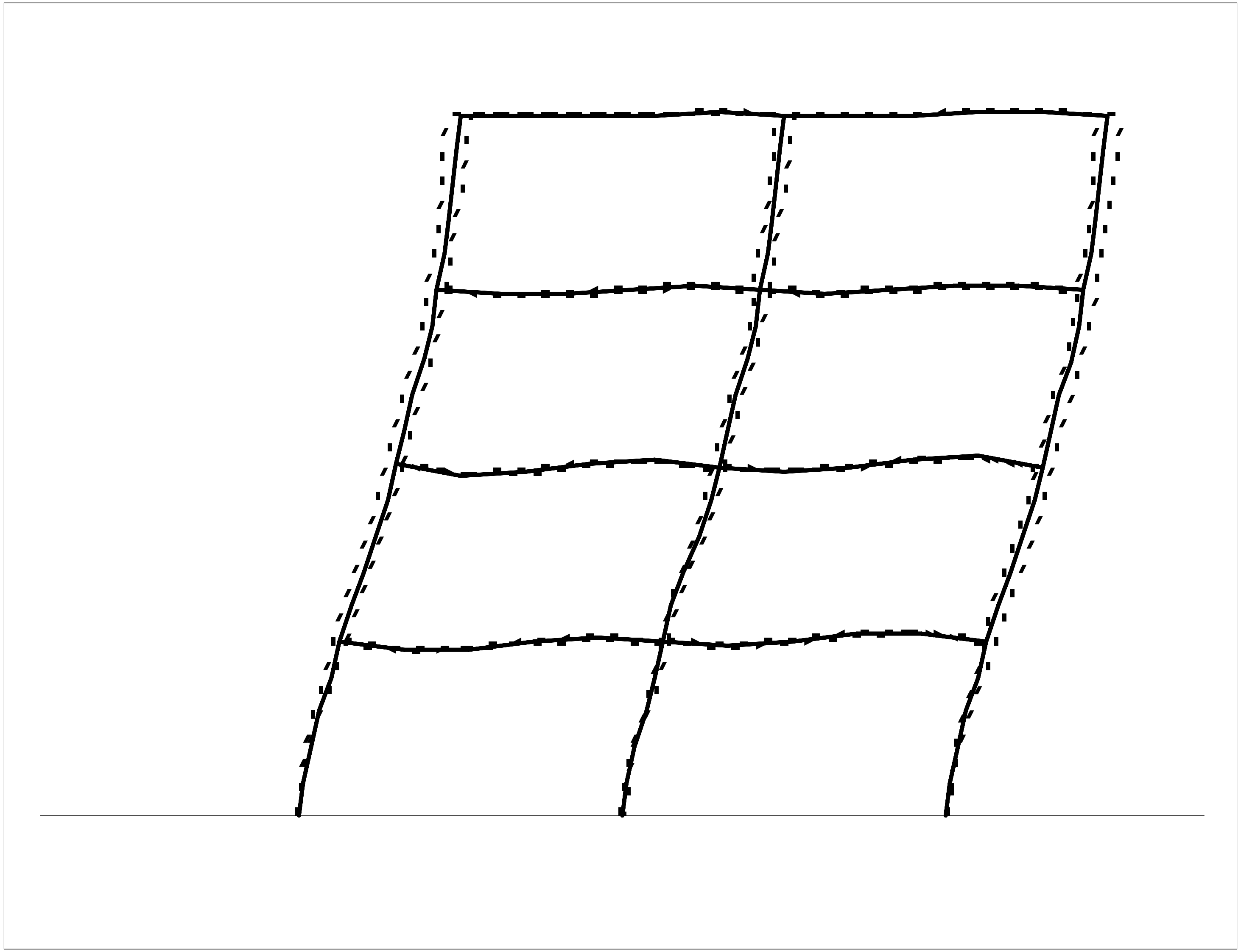


5^{2x4} = **390,625** combinations (deterministic computation runs)

.... negligible improvement in accuracy.

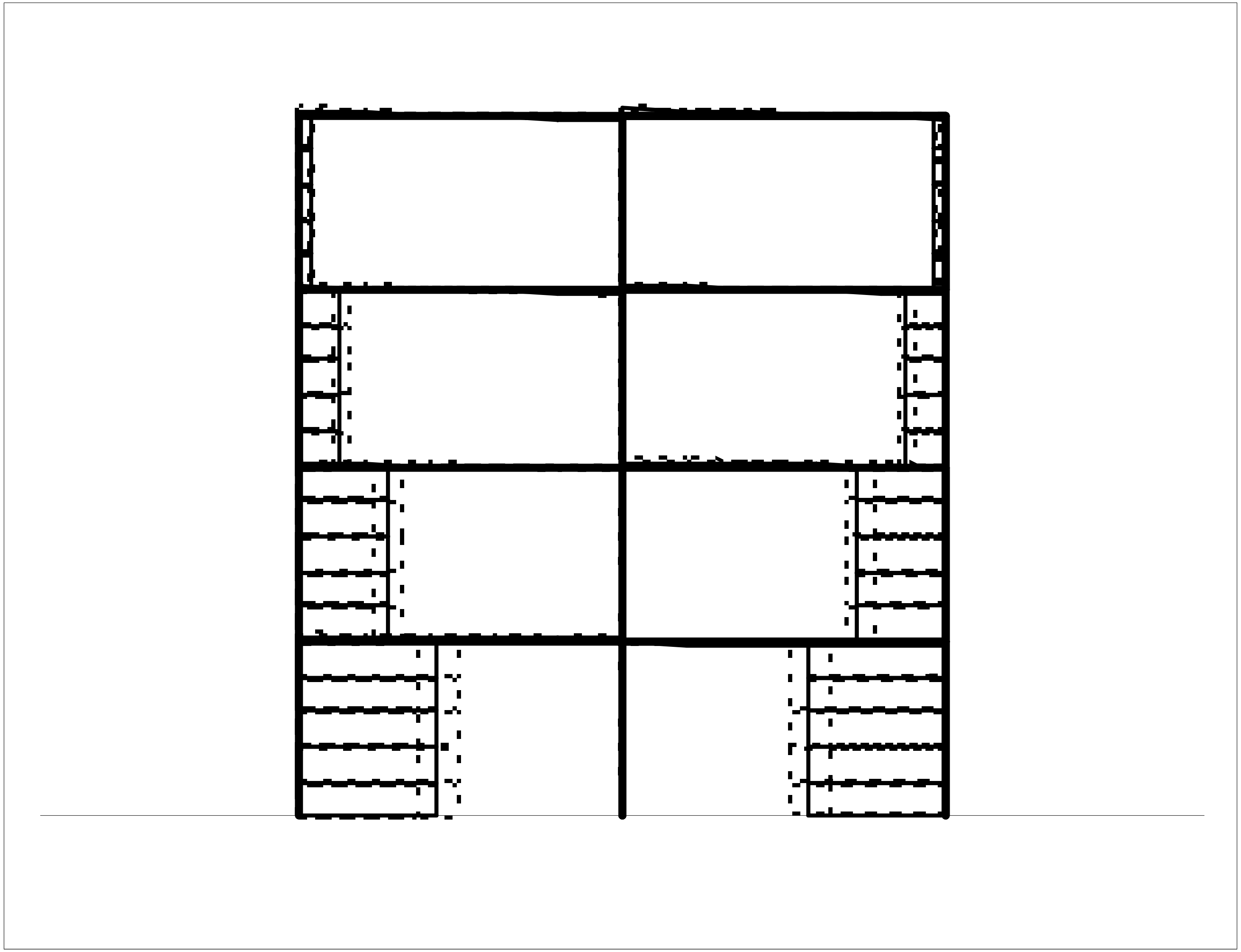
Numerical Example

Results: Distribution of displacements



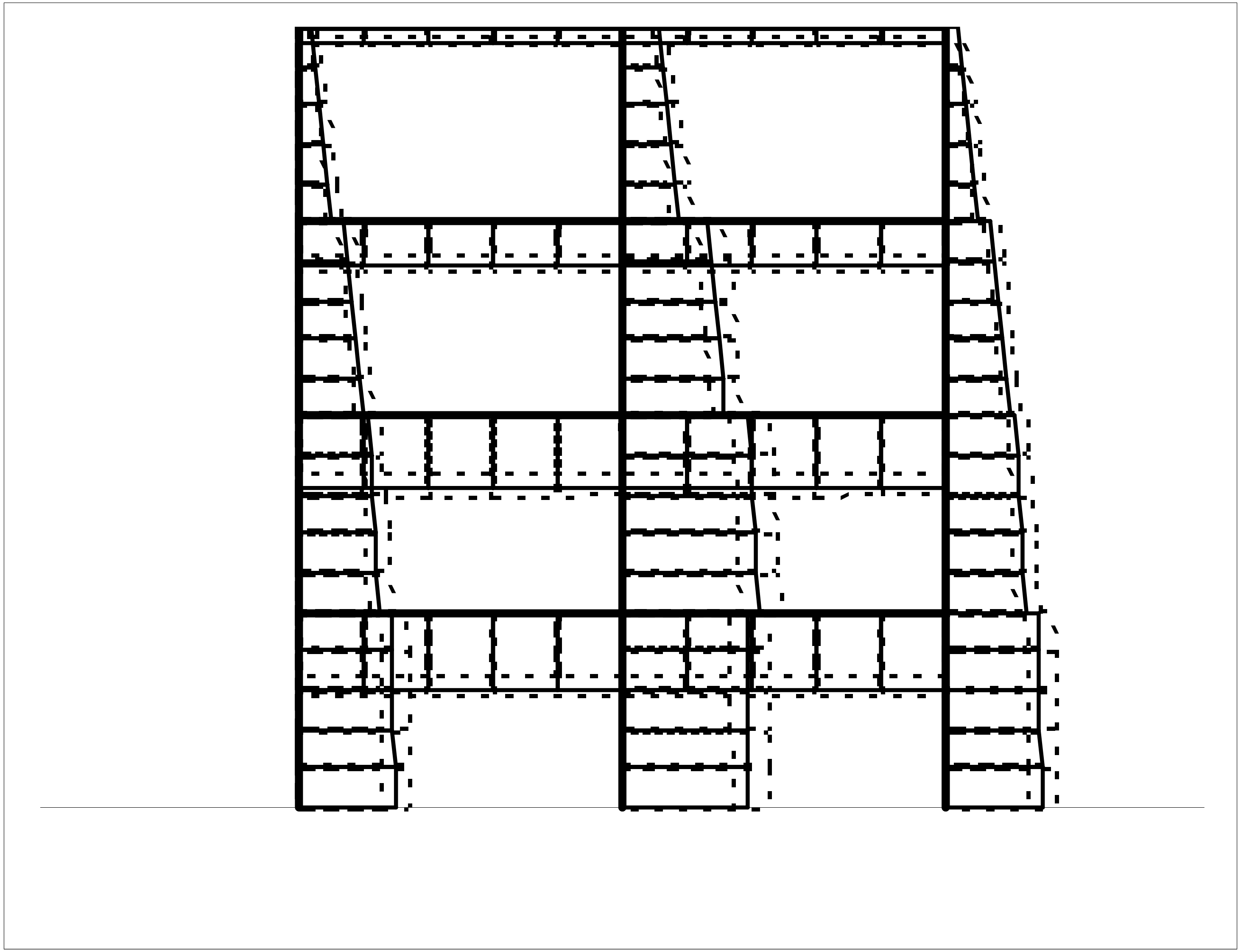
Numerical Example

Results: Distribution of normal forces



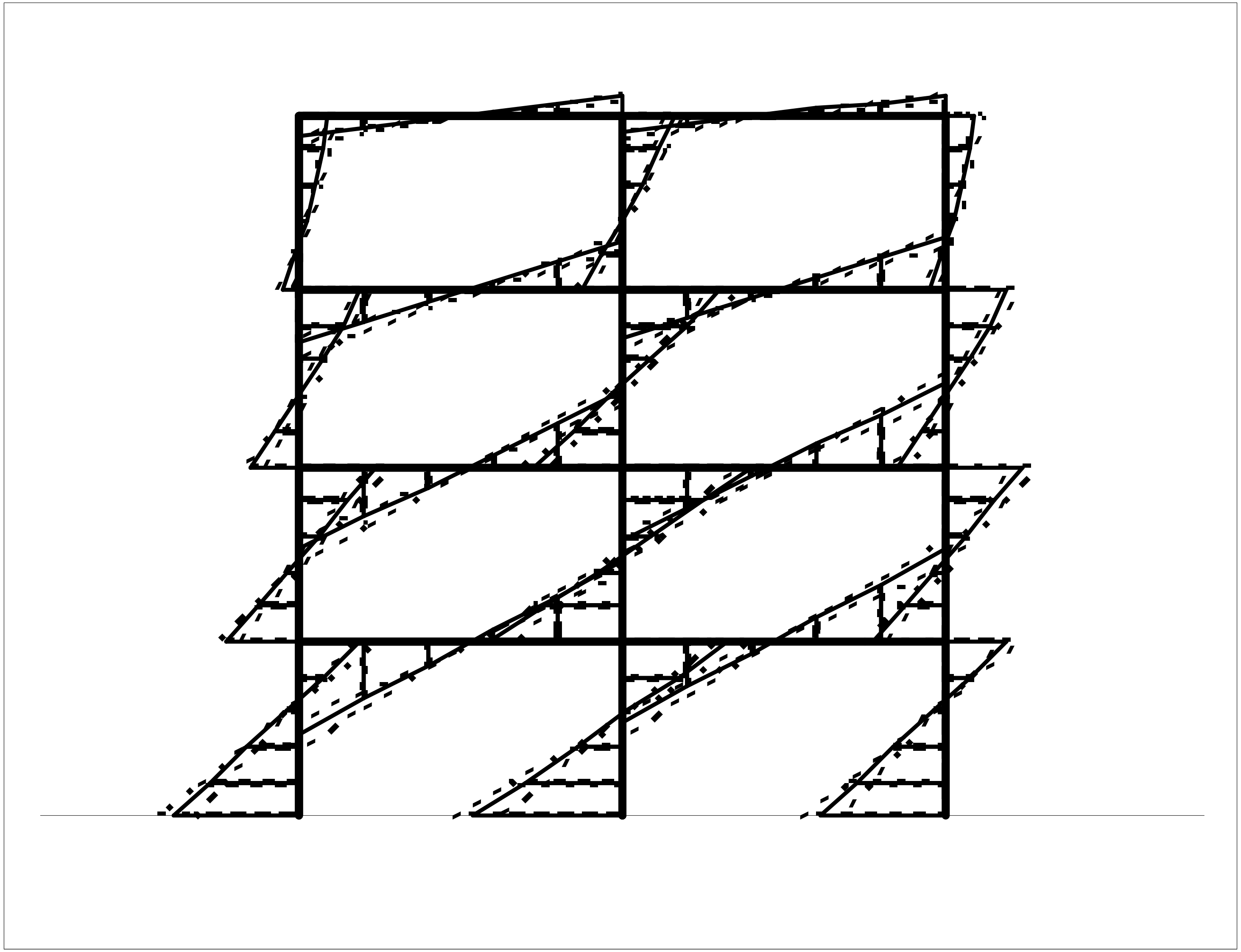
Numerical Example

Results: Distribution of shear forces



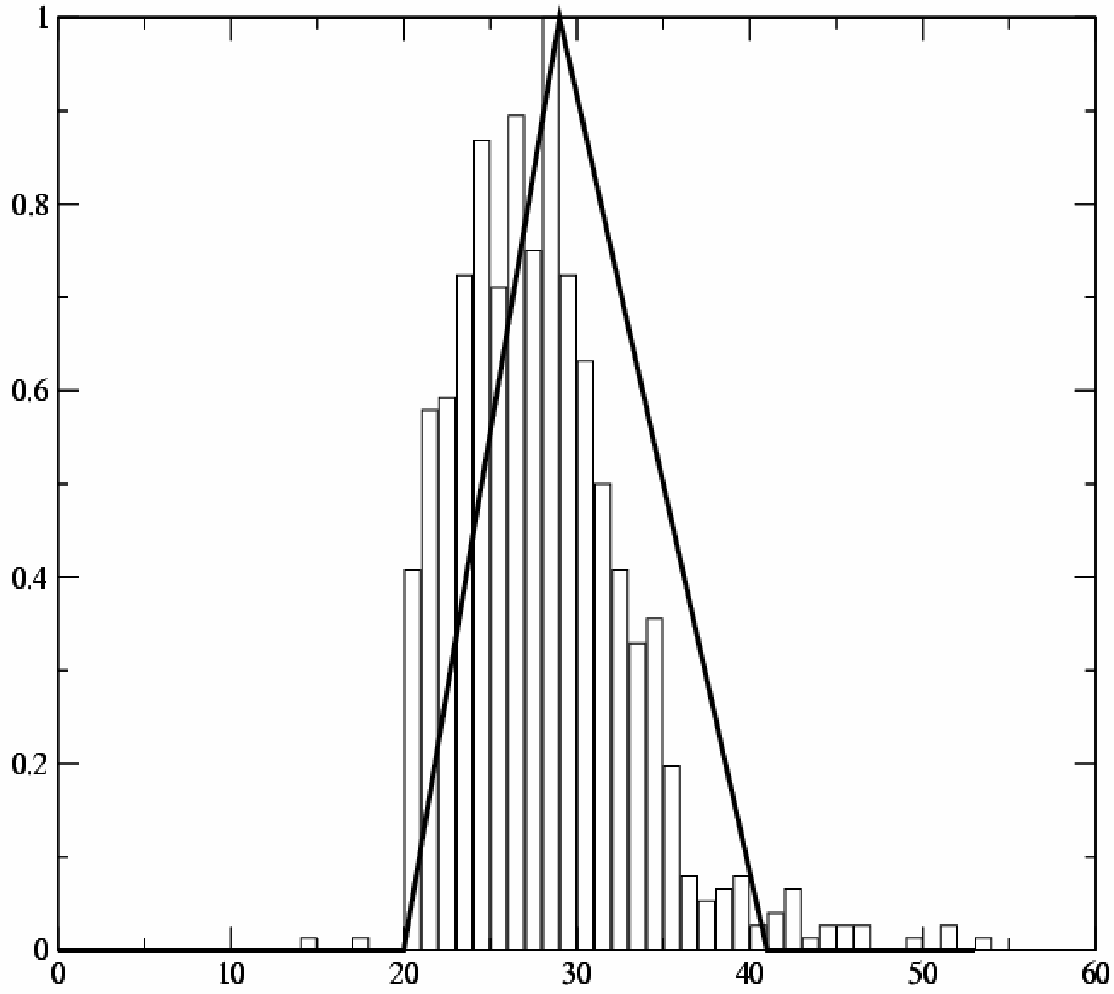
Numerical Example

Results: Distribution of bending moments



Construction of Input Parameters

Compressive strength



Conclusions

1) The concept of fuzzy earthquake design based on response spectrum analysis was shown.

2) Fuzzy dynamic finite element method can be supplemented with the surface response function concept which increases computational efficiency.

3) It is hinted that input and output data collected through combinations of only three values (minimum, modal value, maximum) yield surface response functions with errors up to 5% from true results for dominant responses.

4) This method can serve as a tool for verification that the structural response is within design limits even if the input data contain uncertainty.